

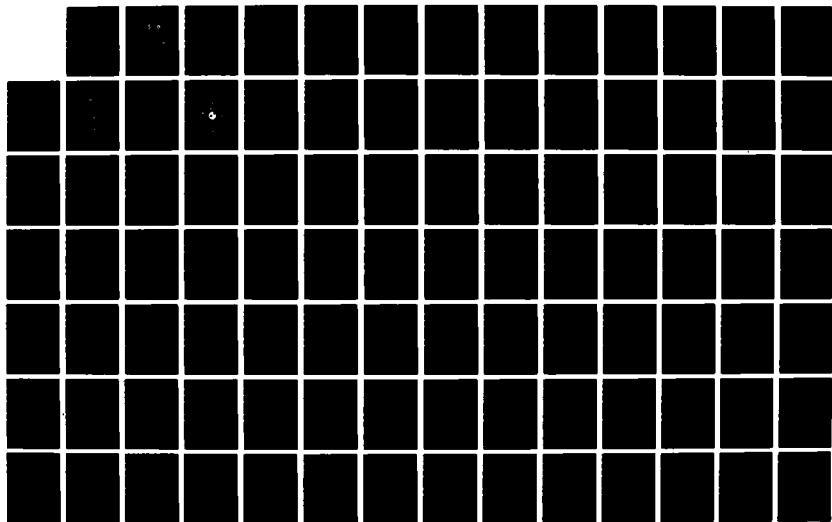
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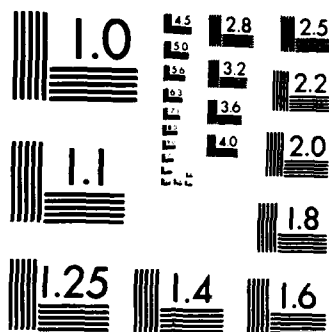
USE OF THE GLOBAL POSITIONING SYSTEM FOR TRAJECTORY  
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USE OF THE GLOBAL POSITIONING SYSTEM

FOR ~~SRAM II~~ TRAJECTORY DETERMINATION ~~OF SRAM II~~ MISSILES

AT THE EASTERN TEST RANGE

THESIS

Edwin A. Zehner  
Captain, USAF

AFIT/GSO/ENS-ENG/86D-2

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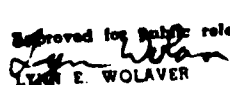
Captain Edwin A. Zehner was born on 23 September 1958 in Ashland, Kansas. He graduated from South Park High School in Fairplay, Colorado in 1976 and entered the Air Force Academy the same year. He received a Bachelor Science degree in Biology upon graduation in 1980. He then attended Initial Qualification Training for the Minuteman III weapon system at the 4315 Combat Crew Training Squadron, Vandenberg AFB, California. This was followed by duty as a missile combat crew member, evaluator, and emergency war order instructor until entering the Air Force Institute of Technology School of Engineering in June 1985.

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This research was intended to determine the accuracy that can be obtained using the Global Positioning System to track SRAM II missiles at the Eastern Test Range. The final goal of the research was to make a definitive calculation of the accuracy offered and compare this against the required accuracy for follow on test and evaluation of SRAM II missiles. The scope was limited to use of GPS equipment currently being developed or already available. The results can be applied to tracking other small, dynamic vehicles at other test ranges.

Aside from the GPS satellite constellation the equipment configuration included a master receive station with a GPS receiver at a surveyed location to support differential calculations, and a translator on board the SRAM II. The GPS signals were to be recorded and subjected to post-test processing for increased accuracy. Using this system all error contributors could be adequately estimated except dynamic error and the error due to multipath. These two errors could be large and must be studied further before a final error level can be confidently stated. Nevertheless, the error level obtained exclusive of these two contributors is certainly low enough to motivate further study of the system.

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USE OF THE GLOBAL POSITIONING SYSTEM FOR SRAM II TRAJECTORY  
DETERMINATION AT THE EASTERN TEST RANGE

THESIS

Presented to the Faculty of the School of Engineering  
of the Air Force Institute of Technology

Air University

In Partial Fulfillment of the  
Requirements for the Degree of  
Master of Science in Space Operations

Edwin A. Zehner, B.S.

Captain, USAF

December 1986

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## Preface

The purpose of this study was to determine whether the Global Positioning System could be used to track SRAM II missiles during follow on test and evaluation flights at the Eastern Test Range. Unfortunately, quantification of two important variables, multipath and dynamics, could not be accomplished in time to be included in the thesis. As a minimum, a boundary of the error introduced by these two variables should be defined before making a final decision concerning whether or not to pursue use of GPS for the stated purpose.

The multipath problem could probably best be solved using a simulation. I feel there are too many variables to approach the problem analytically. The dynamic error can be adequately quantified by further testing the HDLV equipment at the Jet Propulsion Laboratory using the C/A code instead of the P-code.

There are many people who contributed to my effort during this project. I thank my faculty advisors Maj Joe Litko and Dr Darrel Hopper for their assistance and concern. I also wish to thank two outstanding individuals, Lt Wynne Botts and Lt Rick Acosta of the Eastern Space and Missile Center, for their valuable time and effort. And finally, I owe a great deal of thanks to my wife Sherri and daughter Jamie for their patience and support.

The title on the DD Form 1473 is correct  
for this report.  
Per Ms. Verna Graham, AFIT/EN

Ed Zehner



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Abstract

This research was intended to determine the accuracy that can be obtained using the Global Positioning System to track SRAM II missiles at the Eastern Test Range. The final goal of the research was to make a definitive calculation of the accuracy offered and compare this against the required accuracy for follow on test and evaluation of SRAM II missiles. The scope was limited to use of GPS equipment currently being developed or already available. The results can be applied to tracking other small, dynamic vehicles at other test ranges.

Aside from the GPS satellite constellation the equipment configuration included a master receive station with a GPS receiver at a surveyed location to support differential calculations, and a translator on board the SRAM II. The GPS signals were to be recorded and subjected to post-test processing for increased accuracy. Using this system all error contributors could be adequately estimated except dynamic error and the error due to multipath. These two errors could be large and must be studied further before a final error level can be confidently stated. Nevertheless, the error level obtained exclusive of these two contributors is certainly low enough to motivate further study of the system.

USE OF THE GLOBAL POSITIONING SYSTEM  
FOR SRAM II TRAJECTORY IDENTIFICATION  
AT THE EASTERN TEST RANGE

I. Introduction

There is a need for a trajectory identification system for SRAM missiles fired at the Eastern Test Range. In the past the Eastern Space and Missile Center (ESMC) conducted tests of Short Range Attack Missiles (SRAM) using a ground based radar to track the missile and accurately identify its trajectory. Air Force technicians then compared the actual trajectory to the desired trajectory so they could identify any deficiencies in the propulsion and guidance systems. Unfortunately, for reasons not related to the SRAM testing effort, the ground based radar was deactivated. Testing continued for a short time using a ship-born radar but accuracy of this system was so poor that testing was eventually discontinued altogether.

The Global Positioning System (GPS) offers a possible solution. GPS is a constellation of Navstar satellites, each of which continuously transmits a characteristic signal to the earth. Special receivers gather these signals and use them to calculate the location and velocity of the receiver. If such a receiver were coupled to a translator the signals could be gathered and rebroadcast to a ground station where the trajectory of the vehicle carrying the receiver/translator could be calculated and recorded for later study. Such a receiver/translator

package could be placed aboard a SRAM missile in conjunction with an ESMC range upgrade to provide the capability to accurately and reliably track the missile trajectory.

GPS has been studied for range instrumentation but there has been no implementation and no study directly concerning using GPS for a SRAM II test flight tracking system. There is a need to determine if a GPS trajectory identification system will provide sufficient accuracy for effective testing. This research will serve as a foundation for establishing whether or not GPS can be successfully used for this purpose.

This study is limited to using GPS for a tracking system for SRAM II test flights at the Eastern Test Range. However, when the final system is complete it will constitute a generic tracking capability for tests of other weapon systems conducted over any test range. It is further limited to use of off the shelf equipment. No equipment developed specifically for SRAM II tracking is to be required.

#### Background

GPS Overview. GPS will provide position, velocity and time information to users anywhere in the world at any time of the day. The system development is being lead by Air Force Space Division and has drawn the direct interest of all four branches of DOD, the Defense Mapping Agency, the Department of Transportation, and NATO (Wooden, 1984:2) to name a few. The Global Positioning System consists of three segments: user, control, and space as shown in Fig 1. In addition, one must consider a host of other applicable issues to gain a full understanding of GPS. These will be introduced here.

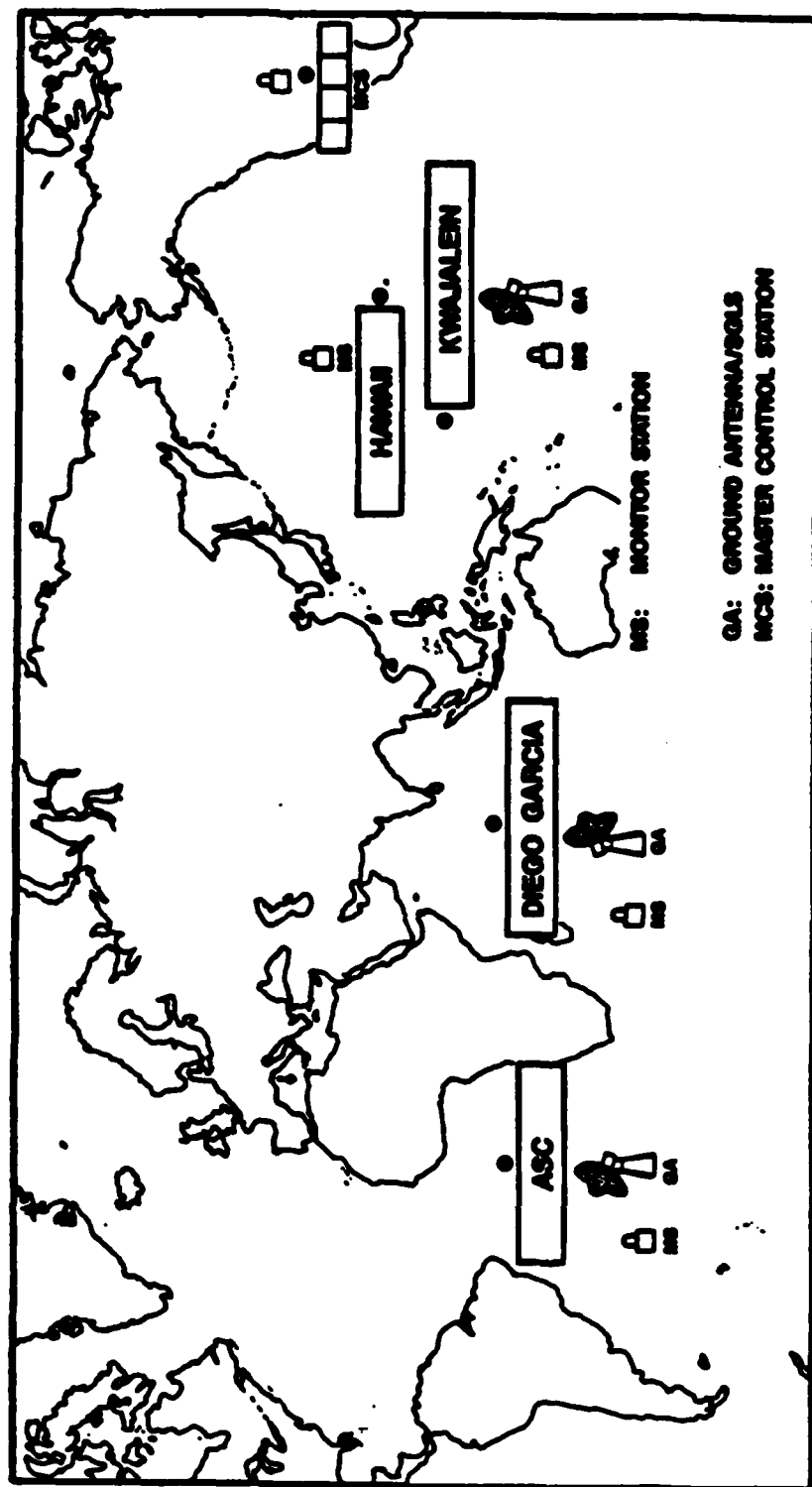


Fig 1. GPS Control Segment



User Segment. The user segment is composed of those equipped to receive the GPS signals and process them into time, position and velocity information. The navigation community, both DOD and civilian, is expected to be the primary user but GPS applications are certainly not limited to this. GPS is being considered for other uses such as a guidance system for tactical missiles (Roemerman, 1981:E9.4.1), a manpack which can be used by foot soldiers to guide tactical troop movements (Blomseth, 1981:E9.1.1), and range instrumentation for tracking weapons during test flights (Arnold, 1983:226).

Control Segment. The control segment, or Operational Control System (OCS), is being developed by IBM Corporation. It consists of a Master Control Station (MCS) located at Falcon Air Force Station in Colorado Springs, Colorado, and three unmanned ground antennas which are located at Ascension Island, Diego Garcia, and Kwajalein. In addition, the OCS includes five monitor stations at Colorado Springs, Ascension Island, Diego Garcia, Kwajalein, and Hawaii. The monitor stations are used to track the distance to each Navstar satellite. This information is sent back to the MCS and used to calculate the satellite orbits and ultimately control the satellites themselves. The communication subsystem is the final component of the OCS and provides the data link between the MCS, the monitor stations and the ground antennas (Francisco, 1984:52).

Space Segment. The third segment is the space segment, the satellite constellation as shown in Fig 2. GPS will use six orbital planes, all inclined at 55 degrees. Each plane will hold three satellites 120 degrees apart and the planes themselves will be separated

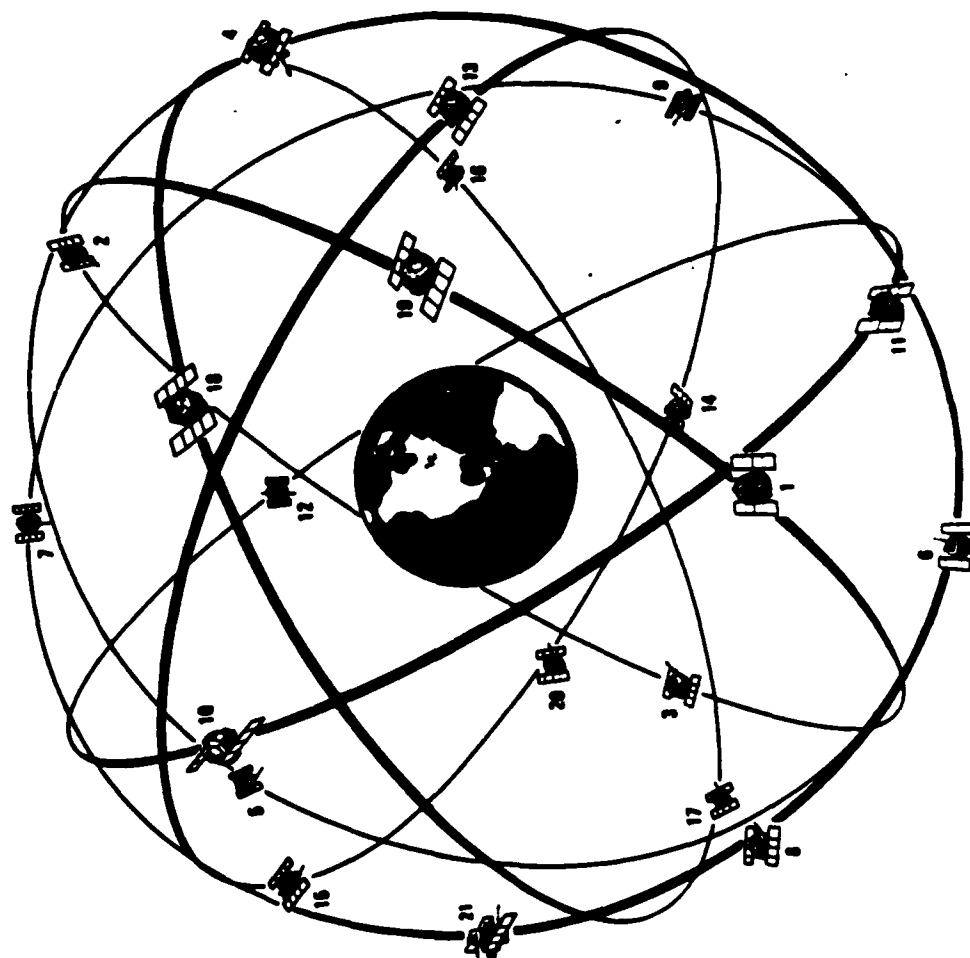


Fig 2. GPS Satellite Constellation

by 60 degrees at their ascending nodes. All satellites will be at an altitude of 20,183 km with a period of 11 hours, 57 minutes, 57.96 seconds (Van Dierendonck, et al, 1980:62).

Signal. The GPS signal characteristics were summarized by McConnell and Pickett of the Western Space and Missile Center.

The GPS satellites continuously broadcast on two L-band frequencies 1575.42 MHz ( $L_1$ ) and 1227.6 MHz ( $L_2$ ). Superimposed on these carriers are two coded signals unique to each satellite: a precision code (P-code) pseudorandom noise (PN) signal with a 10.23 MHz chip rate [bit rate] and a coarse/acquisition code (C/A) PN signal with 1.023 MHz chip rate. The  $L_1$  frequency contains both the P-code and C/A code while the  $L_2$  frequency contains either a P or C/A code. Superimposed on the P and C/A codes are 50 Hz chip rate navigation data containing the navigation message (McConnell and Pickett, 1983:239).

Several of McConnell and Pickett's points require further explanation. First, one reason to have two L-band frequencies is that the second frequency gives the capability to correct for ionospheric delay of the signals. The delay is caused by refraction of the satellite signals as they pass through the ionosphere. Second, the C/A code is less accurate than the P-code (within 55.4 meters as opposed to 15.7 meters) (Arnold, 1983:228). The C/A code is an important feature despite its relative lack of accuracy, because it is much easier to use to establish position and velocity than the P-code. In addition, the C/A code has a signal strength 3 dB higher than that of the P-code. Finally, it is also important to note that the navigation data message is actually the satellite's ephemeris (position descriptors) and its clock parameters (Milliken and Zoller, 1980:7).

Navigation Solution. The GPS provides position and velocity information as follows. The receiver matches the pseudorandom noise code from each of four Navstar satellites with identical internally

generated code streams. The code from a satellite will be shifted somewhat from the internally generated code due to the time necessary to travel from the satellite to the receiver. This is simply a relative phase shift between the two code streams. The amount of shift in the code exactly quantifies the transmission time required for the signal to span the intervening distance. The range, or distance to each satellite from the receiver is, then,

$$R = c(t_R - t_T)$$

where R is the range, c is the speed of light,  $t_R$  is the time the code was processed at the receiver and  $t_T$  is the time the code was sent from the satellite. The receiver clock may not reflect exactly the same time the standard GPS clock does so this is called the pseudorange measurement. Since the receiver is simultaneously processing signals from four satellites, it now has four pseudorange measurements. These are used to solve the four equations in four unknowns, namely, the x, y, and z dimensions of position and the receiver time offset from standard GPS time (Arnold, 1983:227).

For velocity, the GPS receiver once again compares the signal from the satellite to its internally generated signal. This time, however, the receiver checks the doppler shift (apparent shift in frequency) of the incoming signal. The amount of doppler shift is directly related to the velocity of the receiver relative to the satellites (Brooks, 1983:247).

Error Sources. As long as the user can receive four good signals from GPS satellites it can solve for time, position, and velocity. The accuracy of the solution, however, is not exact. Error is introduced

into the solutions from a number of different sources including ionospheric delay of the signal, receiver noise, and high acceleration of the receiver. All error sources must be dealt with individually to determine their impact on this particular application of GPS.

Unfortunately, the errors are not exactly quantifiable. Therefore, an estimation of the error is used. This is called the one sigma UERE or user equivalent range error. This is the expected error at one standard deviation away from a mean of zero. In other words, since the error contributors are independent and the total error is the result from many sources, the distribution of the errors is assumed to be normal. Therefore, the one sigma UERE is an estimate of range error that will be equal to or less than the actual range error about 68% of the time. Since each error source independently contributes its own one sigma UERE, the total estimated error can be obtained by finding the root sum of squares (rss) of all error contributors. Once this figure has been calculated, the actual system accuracy is obtained by multiplying by the dilution of precision factor.

GDOP. GDOP, or geometric dilution of precision, concerns the loss of precision of the measurement for time, position and velocity due to the relative positions of the satellites and the tracked vehicle. The actual calculation of the GDOP is very involved and will be covered in its entirety in a later section.

Accuracy Enhancements. Fortunately, if the accuracy of the basic GPS system is insufficient there are several possibilities for improvement. Two of these are differential navigation and post-test processing. The differential navigation method takes advantage of the

special characteristics of the error sources and GDOP to yield an improvement in overall accuracy of GPS. Post-test processing uses sophisticated filters and smoothers on recorded GPS data to yield a significant increase in accuracy.

### Research Questions

A specific set of questions can now be formulated to facilitate the ensuing research. First, how does the GPS system work? It is necessary to document not only the equipment and facilities involved, but how they interact to provide navigation information. This will lay the groundwork for the second basic research question: what limits the accuracy of the system? In other words, the errors introduced into the navigation information have specific sources, these sources and their characteristics must be identified. Given this, the third question can be posited: how accurate is the GPS system and what can be done to improve this accuracy? The answer to this will lead to the solution of the overall problem and to the answer to the fourth question: what is the expected accuracy of the GPS system when used to track SRAM II missiles? These questions were used to guide the structure of the research, and led to the method of analysis.

### Method of Analysis

The necessary structure behind this study is relatively simple. First, an exhausting research of GPS configurations and potential accuracies must be accomplished and documented. Since the potential accuracy of the system is actually a function of the loss of accuracy due to several error contributors, these contributors must be fully investigated. Once their characteristics are defined, the error

contributors will be put in the context of the problem at hand:

tracking SRAM II missiles at the Eastern Test Range. The result will be an expected accuracy of the GPS system when applied to this problem.

The final task, of course, is to evaluate the expected accuracy of GPS for this particular application and measure it against defined standards. It will then be possible to either reject proposals for further development of GPS use at the Eastern Test Range if the system is not accurate enough, or to continue development and more detailed studies if the system promises to provide sufficient accuracy.

## II. Working Concepts

The working concepts behind the GPS system are detailed in the next four sections. These are the navigation concept, the signal, the navigation message, and the navigation solution. Another concept that is central to the use of GPS is translation. This will be discussed in Chapter V to allow introduction of material that is important to a full understanding of translation.

### The Navigation Concept

The idea behind using GPS signals to find TSPI (time, space, and position information) is a simple one. The receiver gathers a signal from each of four satellites. The location of the satellites is known precisely since this information is part of the signal. The receiver then calculates the distance to all four satellites. Given the range to and location of each satellite the receiver can find the one point where the four ranges from the four satellite locations can intersect, this is the position of the receiver. To measure the velocity of the receiver, it simply measures the doppler shift of the incoming signals.

A detailed description of the method follows. There are many sources for error with this technique. This error limits the system from achieving pinpoint accuracy but is not so large as to render the system useless. These sources of error will be covered in a later section.

The basic problem of finding time and position can be broken down into finding four unknowns: position coordinates  $x$ ,  $y$ ,  $z$ , and time  $T$ . When the receiver processes a signal from the satellite it actually



matches a code on the signal generated by the satellite with an exact copy of the code generated in the receiver itself. The code in the signal sent from the satellite will be phase shifted a small amount relative to the internally generated code due to the time required for the signal to travel from the satellite to the receiver. Since the receiver can measure this shift it can find the signal transit time and therefore the distance to the satellite since the speed with which the signal travels is known.

Unfortunately, the measured range is only an estimate of the actual range and so is called the "pseudorange". The range equation is outlined in an article by Van Dierendonck, et al (1980:55-73). The actual range,  $R_{SUi}$ , between satellite  $i$  and the receiver is

$$R_{SUi} = c(t_R - t_{Ti}) - ct_{di}; \quad i=1, \dots, 4$$

where

$R_{SUi}$  = the range to the  $i^{th}$  satellite

$c$  = the speed of light

$t_R$  = the GPS receive time

$t_{Ti}$  = the GPS transmission times

$t_{di}$  = the known propagation delays

However, the system does not deal with the actual range since  $t_{di}$  is not known exactly and since the GPS time clocks cannot be perfectly synchronized. Therefore the system uses an approximation yielding pseudorange as noted above. This can be calculated using

$$R_i = R_{SUi} + ct_{di} + \Delta t_{di} + c(T_U - T_{Si}); \quad i=1, \dots, 4$$

where

$\Delta t_{di}$  = unknown propagation delays

$T_U$  = user clock offset

$T_{Si}$  = satellite clock offset

Since the true range can be calculated as the rss (root sum of squares) of the differences between the receiver position and the satellite positions

$$R_i = [(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2]^{\frac{1}{2}} + ct_{di} + \Delta t_{di} + c(T_U - T_{Si}); \quad i=1, \dots, 4$$

where

$x_i, y_i, z_i$  = coordinates of the  $i^{\text{th}}$  satellite position

$x, y, z$  = coordinates of the user position

Solving for the receiver's position involves setting this equation up for each of the four satellites. This gives four equations in four unknowns.  $x_i, y_i$ , and  $z_i$  are all known since the navigation message includes satellite ephemeride prediction parameters.  $T_{Si}$  is also transmitted in the satellite signal and so is known.  $t_{di}$  is found by solving the pseudoranges using two separate frequencies. This allows the receiver to solve for ionospheric delays of the signal. In addition, geometric models can be used to approximate delays in the troposphere. These delays are included in the  $t_{di}$  term. The remaining unknown delays are represented by the  $\Delta t_{di}$  term and will result in errors in the range measurement. The only other unknowns are the  $x, y, z$  terms and  $T_U$ . Note that if the receiver is equipped with an accurate

clock in GPS time, only three satellites are required to solve for TSPI since  $T_U$  would then be known (Sturza, 1984:122-132). The actual navigation equations used to solve for TSPI in a four satellite scenario will be detailed in a later section.

The velocity of the receiver is measured via the doppler shifts of the satellite signals. The change in range from some time  $t$  to an earlier time  $t_0$  is

$$R(t) - R(t_0) = \lambda[\phi(t_0, t) - \phi(t_0', t')] \quad (1)$$

given that  $\lambda$  is the wavelength of the carrier,  $\phi(t_0, t)$  is the number of cycles occurring during the interval from  $t_0$  to  $t$  in the internally generated signal while  $\phi(t_0', t')$  is the number of cycles in the signal from the satellite during an interval from  $t_0'$  to  $t'$  (Brooks, 1983:247). The two time intervals are of exactly the same length but occur at different times due to the transit time of the signal from the satellite.

The velocity, then, is calculated as the change in range over some time interval divided by the magnitude of the time interval. Note that the velocity component obtained is along the line of sight between the satellite and the receiver. The doppler shift must be measured from three satellites to get true three dimensional velocity.

### The Signal

The Navstar navigation signal is actually a composite of several signals. The Navstar signal is broadcast on two frequencies, both of which are even multiples of the satellite central clock frequency standard of 10.23 MHz. The first frequency, called Link 1 or L1, is

1575.42 MHz ( $154 \times 10.23$ ). The second frequency, L2, is 1227.6 MHz ( $120 \times 10.23$ ). Both the L1 and L2 frequencies carry a navigation message which will be detailed later. In addition, the L1 frequency carries two pseudorandom noise (PN) codes. The L2 signal, on the other hand, is modulated only with the P-code or the C/A code at any one time (Milliken & Zoller, 1980:6).

The first of these PN codes is the P-code, so named because it yields a more precise identification of time, space, and position information (TSPI). The second code is the C/A code or coarse/acquisition code. This is also a PN code and yields a less accurate estimate of TSPI but is much easier to acquire and allows the receiver to solve for TSPI much more quickly. Generally, the receiver-processor will acquire the C/A code first and then use information gleaned from this code to switch over to the P-code for more accurate information.

The P-code is a product of two PN code generators, X1 and X2. According to Spilker (1980:38) the period of the X2 generator is 15,345,037 bits and the period of the X1 generator is 15,345,000 bits. The X1 generator returns to its initial state every 1.5 seconds since the chip rate is 10.23 MHz ( $15,345,000/10.23 \times 10^6 = 1.5$ ), and the X2 generator runs for  $37/10.23 \times 10^6$  seconds longer. Since the periods of the two generators are relatively prime, the overall period of the P-code is a little more than 38 weeks. This time period is divided up with each satellite using a different week of the code. Therefore, every satellite is using a code that is unique to it. This is used to distinguish the satellites from one another.

The C/A code is much shorter than the P-code. It is 1023 bits long and requires only 1 msec to complete at 1.023 Mbps (Spilker, 1980:38). The utility of using a short period for this code is in the acquisition time for the receiver (Milliken & Zoller, 1980:6). The receiver must match its internally generated codes with that of the incoming signal. Far too much time is required to search a one week PN stream such as the P-code. Since the period of the C/A code is only 1 msec, the receiver can search the code and match it much more quickly. With this match successfully completed, the receiver reads information from the navigation message called the handover word (HOW). This HOW word changes every six seconds and indicates what point in the P-code stream the P-code will be at when the next HOW word change occurs. The receiver can then lock on to the P-code at the next change and exactly match its internally generated P-code with that of the incoming signal.

Of course, the incoming signal has undergone some changes in phase due to delays in the transmit equipment, the receive equipment and in the channel. As outlined earlier, the receiver models the errors to find the phase change due solely to time required for the signal to reach the receiver from the satellite. When this phase difference is known, it can be translated into the range or distance between the satellite and receiver.

As mentioned earlier, this is not the only information available from the signal. The signal also contains a separate navigation message.

### The Navigation Message

The navigation message is a 1500 bit stream of information which is used by the receiver-processor to gather TSPI. According to Milliken & Zoller this information contains satellite status, a synchronization hand over word (HOW), parameters for clock correction, satellite ephemerides, atmospheric propagation delay corrections, and ephemerides and status of all other GPS satellites in the constellation (1980:7). The 1500 bit stream is organized in a 30 second frame which is divided into five subframes of six seconds in length.

Spilker (1980:40) documented the use of the individual subframes which is shown in Fig 3. Each subframe begins with a TLM or telemetry word which is used by the receiver to acquire the message (Milliken & Zoller, 1980:7). The next portion of each subframe is the HOW, the use of which was described in the last section. Finally, each subframe contains a different block of information.

Block 1 contains the clock correction parameters. It is important that the satellite clocks are all synchronized. Recall that the pseudorange measurement is time dependent. The receiver calculates the

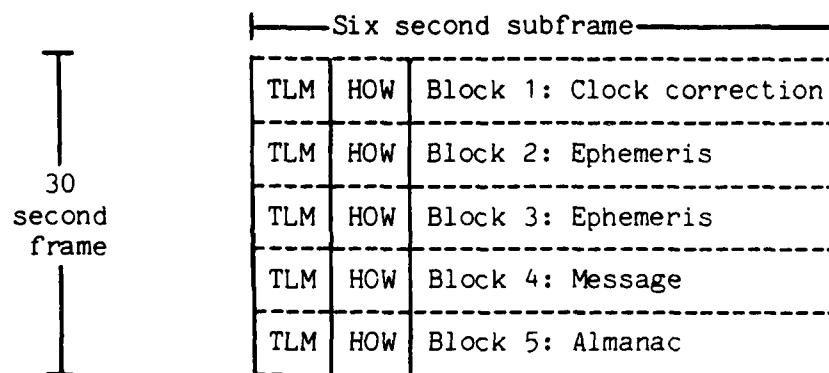


Fig 3. GPS Data Frame (Spilker, 1980:40)

time required for the signal to travel from the satellite. If the time the signal was sent is not exactly known, no measure of range can be made. As an example of the time accuracy required, consider a time error of ten nanoseconds. In this miniscule period of time the signal will travel  $(3 \times 10^8 \text{ m/s}) \times (10 \times 10^{-9} \text{ s})$  or 3 meters. This is on the order of the desired accuracy of the system, therefore the clock errors must be much smaller.

Actually, the satellite clocks are allowed to deviate from GPS standard time by as much as 976 microseconds (Milliken & Zoller, 1980:5). This deviation is measured by the MCS and its magnitude is sent to the user in the clock error correction data in Block 1 of the navigation message. The receiver automatically compensates for the deviation when it calculates the pseudorange to the satellite. The actual error in the clock which is not measurable by the MCS is on the order of one nanosecond. To achieve this accuracy, atomic clocks are used (Milliken & Zoller, 1980:5).

Data Block 1 also contains 8 bits of information which reflect a modelled value of atmospheric delay of the signal. This is used by receivers requiring less accuracy in TSPI resolution. Such receivers normally are equipped to receive only the L1 signal and could not otherwise calculate the atmospheric delay (Milliken & Zoller, 1980:7). Note that if the atmospheric delay is not removed from the total time delay during signal travel, the pseudorange measurement would be too long.

Data Blocks 2 and 3 are in subframes 2 and 3 and contain the satellite ephemeris (Spilker, 1980:40). To be more accurate, this information actually consists of the satellite ephemeris prediction

parameters. These parameters are updated each hour by the MCS (Milliken & Zoller, 1980:7). The receiver must use the prediction parameters to calculate the exact location of the satellite at the time of the transmission.

Data Block 4 is in subframe 4 and is used to send any special messages to the receivers from the MCS.

Data Block 5 is in the final subframe and is the almanac of information pertaining to the other satellites in the constellation. This information includes ephemeris predictors, clock correction parameters and atmospheric delay correction parameters for the other satellites. There is more information than can fit in one subframe so the almanac information is continued in subframe 5 of the next five frames. Thus information for the entire constellation is in a total of six frames with subframe 5 rotating to a "new page" with each frame. Making this information available to the user allows easier acquisition of the other satellites (four are required for complete TSPI). The receiver uses the information to run the ephemeris of the other satellites in view through an algorithm which finally yields which four should be used for the most accurate fix (Milliken & Zoller, 1980:8). It is, of course, not required that a receiver wait until all six frames are passed before it goes on to another satellite.

The totality of this information provides the necessities for finding TSPI using GPS. All the information needed to find the navigation solution are contained in the navigation message. The navigation solution, however, is not actually contained in the five subframes, but must be calculated from it.



### The Navigation Solution

The basic technique for obtaining TSPI was detailed in the Navigation Concept section. The following is a more precise and rigorous discussion yielding the TSPI solution. This includes the derivation and processing of the set of linear equations which finally give the user position and can ultimately be used to calculate the error in the position information. This information is extracted directly from a report titled "Normalized Accuracy Analysis of the Navstar/GPS" written by P. S. Jorgensen. Mr. Jorgensen was with the Systems Engineering Operations section of The Aerospace Corporation in 1978 when the report was published.

The information generally needed by the user is the exact position on the face of the earth. This position can best be described using an earth centered cartesian coordinate system. The x axis extends through the intersection of the equator and the Greenwich Meridian. The z axis extends through the north pole and the y axis completes the right handed system. The distance between two points in a cartesian system can be calculated as the root sum of squares (rss) of the differences in the positions of the points on the respective axes. A complicating factor which must be added in the case of the GPS system, however, is the user clock offset. The range from a satellite to a user would then be

$$R_i = [(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2]^{1/2} + T$$

where

$R_i$  = pseudorange to the  $i^{\text{th}}$  satellite

$x, y, z, T$  = user position and clock offset range equivalent

$x_i, y_i, z_i$  = position of the  $i^{\text{th}}$  satellite

Note  $R_i$  is called the pseudorange as opposed to actual range due to the unknown user clock offset. It should also be noted that the units for the clock offset  $T$  are not time units but instead are distance. The time is here represented by its range equivalent, the distance travelled in the elapsed time. This is found by multiplying the time (clock offset) by the speed of light.

Note also that the errors introduced into the system from sources such as atmospheric delay are not included in this solution. These quantities must be estimated and removed as necessary once the basic solution is found.

There are now four of the above equations, one for each of the four satellites to be used in the solution ( $i=1,2,3,4$ ). The known quantities are the satellite positions, while the user position and clock offset are unknown. There, are then, the familiar four equations and four unknowns. These equations can be simplified by making them linear. This is necessary so the solution method is compatible with prospective user equipment. To this end let

$x_n, y_n, z_n, T_n$  = nominal (best estimate) values for the actual position and user clock offset ( $x, y, z, T$ )

$\Delta x, \Delta y, \Delta z, \Delta T$  = difference between the actual and nominal values of  $x, y, z$ , and  $T$

$R_{ni}$  = nominal pseudorange measurements to the  $i^{\text{th}}$  satellite

$\Delta R_i$  = difference between the actual and nominal values of  $R$

Note  $R_i$  is called the pseudorange as opposed to actual range due to the unknown user clock offset. It should also be noted that the units for the clock offset  $T$  are not time units but instead are distance. The time is here represented by its range equivalent, the distance travelled in the elapsed time. This is found by multiplying the time (clock offset) by the speed of light.

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$R_{ni}$  = nominal pseudorange measurements to the  $i^{\text{th}}$  satellite

$\Delta R_i$  = difference between the actual and nominal values of  $R$

Altogether, then

$$x = x_n + \Delta x$$

$$y = y_n + \Delta y$$

$$z = z_n + \Delta z$$

$$T = T_n + \Delta T$$

$$R_i = R_{ni} + \Delta R_i$$

$$R_{ni} = [(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2]^{\frac{1}{2}} + T_n$$

Substituting these six relationships into the basic pseudorange equation gives

$$\begin{aligned} & [(x_n + \Delta x - x_i)^2 + (y_n + \Delta y - y_i)^2 + (z_n + \Delta z - z_i)^2]^{\frac{1}{2}} \\ & = R_{ni} + \Delta R_i - T_n - \Delta T \end{aligned}$$

By doing a first-order expansion the relationship becomes

$$\begin{aligned} & [(x_n - x_i)^2 + (y_n - y_i)^2 + (z_n - z_i)^2]^{\frac{1}{2}} \\ & + \frac{(x_n - x_i)\Delta x + (y_n - y_i)\Delta y + (z_n - z_i)\Delta z}{[(x_n - x_i)^2 + (y_n - y_i)^2 + (z_n - z_i)^2]^{\frac{1}{2}}} \\ & = R_{ni} + \Delta R_i - T_n - \Delta T \end{aligned}$$

Finally, substituting to eliminate the extra  $(R_{ni} - T_n)$  terms and rearranging

$$\frac{(x_n - x_i)}{(R_{ni} - T_n)} \Delta x + \frac{(y_n - y_i)}{(R_{ni} - T_n)} \Delta y + \frac{(z_n - z_i)}{(R_{ni} - T_n)} \Delta z + \Delta T = \Delta R_i \quad (1)$$

This set of four linear equations provide the solution for TSPI. The unquantified variables are  $\Delta x$ ,  $\Delta y$ ,  $\Delta z$ , and  $\Delta T$ .  $\Delta R_i$  is provided through the GPS signal ranging routine which consists of matching the PN codes from the satellite and the internal generator and checking for phase delay, then scaling the delay time by the speed of light as detailed earlier. The variables  $x_n$ ,  $y_n$ , and  $z_n$  are based on the user's best estimate of current position and  $T_n$  is from a user estimate of clock offset.  $R_{ni}$  is known from the user current position estimate and  $x_i$ ,  $y_i$  and  $z_i$  which are calculated from satellite ephemerides given in the GPS signal.

Possibly leading to a better understanding of the equations, consider the fact that the coefficients of the variables  $\Delta x$ ,  $\Delta y$ , and  $\Delta z$  in Equation (1) are actually direction cosines between the user-satellite line of sight and the respective axes. This holds true because  $(x_n - x_i)$ , for example, is the projection of a vector of length  $(R_{ni} - T_n)$  onto the x axis.

These four equations can be organized in matrix notation. The first of these matrices includes the coefficients of the variables on the left side of the equations. The second matrix is the quantities to be computed in the final solution, the corrections to the nominal values of position and time. The third and final matrix contains the  $\Delta R_i$  terms.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & 1 \\ a_{21} & a_{22} & a_{23} & 1 \\ a_{31} & a_{32} & a_{33} & 1 \\ a_{41} & a_{42} & a_{43} & 1 \end{bmatrix} \times \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta T \end{bmatrix} = \begin{bmatrix} \Delta R_1 \\ \Delta R_2 \\ \Delta R_3 \\ \Delta R_4 \end{bmatrix}$$

where, for example,  $a_{11} = (x_n - x_1)/(R_{n1} - T_n)$ .

For ease of expression, let A be the first matrix, x the second matrix, and r the third matrix. The entire set of equations can now be written as

$$A x = r \quad \text{or} \quad x = A^{-1} r$$

since x is the final solution we seek. Solving for x using conventional matrix algebra methods will yield the user position and time.

There is another further use for these equations arranged in matrix expression. As will be further discussed in the System Accuracy section, the relative arrangement of the four satellites and the user will effect the accuracy of the solution. Specifically, any loss of accuracy due to factors such as atmospheric delay will be multiplied to a larger value by a variable which is dependent upon the relative positions of the user and .pa satellites. This variable is the dilution of precision (DOP) and can be derived using the framework provided in the above equations.

To begin working through this derivation, first recall that  $Ax=r$  is a linear relationship. Therefore, it can also reflect the relationship between the pseudorange measurement errors on the one hand and the user position and clock offset errors on the other.

$$e_x = A^{-1} e_r$$

where

$e_x$  = error in user position and clock offset derived from the  
pseudorange measurement error

$e_r$  = pseudorange measurement error

Since the magnitude of the errors is not known exactly, the errors will be assigned a value based on experimentation and theoretical research. There will of course be a variance associated with each of these values. Now construct covariance matrices consisting of the expected values of the products of the errors.

$$\text{COV } (r) = E [e_r e_r^T]$$

The diagonal terms are the squares or the variances of the expected errors and the off-diagonal terms are the covariances between pseudorange measurements.  $E []$  indicates the expected value of the term.

$$\text{COV } (x) = E [e_x e_x^T]$$

Here the diagonal terms are the variances in the error in user position and time, and the off-diagonal terms are the covariances. The two covariance matrices are related by the equation

$$\text{COV } (x) = A^{-1} \text{COV } (r) A^{-T} \quad \text{or}$$

$$\text{COV } (x) = [A^T \text{COV } (r)^{-1} A]^{-1}$$

Now, according to Jorgensen, these equations can be greatly simplified by two assumptions. Both assumptions are based on the definition of the geometric DOP or GDOP. They are first that the pseudorange measurements have a one sigma error of one when the mean is normalized to zero. Second, the errors received from the pseudorange measurement from one satellite are independent of those received from all other satellites.

In terms of the above equations, these two assumptions dictate that COV (r) is an identity matrix.

Under these circumstances

$$\text{COV (x)} = (\text{A}^T \text{A})^{-1} \quad \text{or}$$

$$\text{COV (x)} = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{xz} & \sigma_{xT} \\ \sigma_{yx} & \sigma_y^2 & \sigma_{yz} & \sigma_{yT} \\ \sigma_{zx} & \sigma_{zy} & \sigma_z^2 & \sigma_{zT} \\ \sigma_{Tx} & \sigma_{Ty} & \sigma_{Tz} & \sigma_T^2 \end{bmatrix}$$

Finally, by definition

$$\text{GDOP} = (\text{TRACE} [(\text{A}^T \text{A})^{-1}])^{\frac{1}{2}}$$

which is the square root of the sum of the diagonal terms in COV (x).

The GDOP, incidentally, is the most encompassing of the five DOP values.

As will be detailed in the System Accuracy section the other terms include the

$$\text{PDOP (Position Dilution of Precision)} = (\sigma_x^2 + \sigma_y^2 + \sigma_z^2)^{\frac{1}{2}}$$

$$\text{HDOP (Horizontal Dilution of Precision)} = (\sigma_x^2 + \sigma_y^2)^{\frac{1}{2}}$$

$$\text{VDOP (Vertical Dilution of Precision)} = (\sigma_z^2)^{\frac{1}{2}}$$

$$\text{TDOP (Time Dilution of Precision)} = (\sigma_T^2)^{\frac{1}{2}}$$

At this point, all the necessary working concepts have been detailed. The two transmitted frequencies, L1 and L2, were reviewed, as well as the navigation message and the two PN codes carried on L1 and L2. The navigation message describes where the satellites are, while



matching the PN codes from the satellites with internally generated codes yields the ranges between the receiver and the satellites. This information can be fed into the navigation solution equations to yield user position. Furthermore, the equations can be used to find the DOP, which is a determinant of how accurate the solution will be and depends upon the relative geometry of the satellites and the receiver. The function of the DOP will be further described in Chapter IV, which is concerned with the accuracy of the GPS system. First, however, the sources of errors that contribute to the loss of accuracy in the system will be detailed in Chapter III.

### III. Error Sources

As mentioned earlier, there are several sources of error in the accuracy of the TSPI from the GPS network. The amount of error each source is allowed to contribute to the total inaccuracy is limited and defined by the GPS specification. These error sources are summarized in Table I. The error sources are independent of one another so the total error is the root sum of squares (rss) of the individual contributors. Equivalent range is used here because the errors result in some change in the time of arrival of the signal from the satellite. This time difference, when scaled by the speed of light, represents an effective difference in the range measurement. Since this range measurement is easier to understand, equivalent range is used instead of the time difference.

The actual numbers in Table I are from the system specification. They can be, and often are, combined or broken out to suit the needs of the author. For example, the same specification used by Milliken and Zoller was used in the preparation of TR 82-2. Yet the GPS System Error Specification Budget in TR 82-2 is slightly different from that used by Milliken and Zoller. The TR 82-2 budget is shown in Table II. Both are dealing with essentially equivalent accuracies in as much as the one sigma UERE is concerned. Note, however, that in Table II the atmospheric delays are broken out into ionospheric and tropospheric delay compensations. Note also that Milliken and Zoller chose not to treat ephemeris prediction and model implementation errors explicitly. Nevertheless both sources give a reasonable estimate of errors

TABLE I

P-Code Range Error Budget (Milliken and Zoller, 1980:9)

Error Source	User Equivalent Range Error (1 sigma) meters
SV clock errors and ephemeris errors	1.5
Atmospheric delays	2.4-5.2
Group delay	1.0
Multipath	1.2-2.7
Receiver noise and resolution	1.5
RSS	3.6-6.3

TABLE II

TR 82-2 P-Code Range Error Budget (1982:2.0-5)

Error Source	User Equivalent Range Error (1 sigma) meters
SV clock and ephemeris errors	2.7
Group delay	1.0
Ephemeris prediction and model implementation	2.5
Ionospheric delay compensation	2.3
Tropospheric delay compensation	2.0
Receiver noise and resolution	1.5
Multipath	1.2
Other	0.866
RSS	5.3

contributed from the several sources. As more information is brought to bear on the subject, these estimates can be refined as will be shown in the ensuing GPS real time and GPS post-test error budgets. First, however, the error sources themselves must be discussed.

The range errors and their discussion here are drawn from an article by Milliken & Zoller published in Navigation (1980:3-14) unless stated otherwise.

### SV Clock Errors

The first error is the satellite vehicle (SV) clock errors. As mentioned earlier, the time dependency of the pseudorange measurement requires that all satellites have their clocks synchronized. Exact synchronization is not possible but the SV clocks are not allowed to deviate from true GPS time by more than 976 microseconds. The magnitude of this deviation is measured by the MCS and a clock correction parameter is included in Block 1 of the navigation message. The receiver uses this parameter to remove this clock offset from the range measurement.

Van Dierendonck, et al, (1980:55-73) gives an excellent explanation of SV clock and GPS time considerations. The following is extracted from their work.

To find GPS time the user must apply

$$T = T_{sv} - T_{Si}$$

where

$T$  = GPS transmission time for the  $i^{th}$  satellite

$T_{sv}$  = SV time at the time of transmission

$T_{Si}$  = offset between SV time and GPS time

Note that  $T_{Si}$  is not constant. This variable changes with time due to oscillator drift and relativistic effects on the SV clock. The changes can be adequately described with the second order polynomial

$$T_{Si} = a_0 + a_1(T - T_{Oc}) + a_2(T - T_{Oc})^2$$

The coefficients  $a_0$ ,  $a_1$ , and  $a_2$  are calculated by the MCS and are included in Data Block 1 as explained earlier. The variable  $T_{Oc}$  is the Data Block 1 reference time. Since this equation is relatively insensitive to the value for  $T$  used,  $T$  can be replaced by  $T_{SV}$  without appreciably degrading its accuracy.

Employing these equations the user can effectively remove SV clock drift and relativistic effects from the SV time. Aside from this the MCS is limited to about one nanosecond in the clock offset it can correct so the overall inaccuracy due to SV clock errors is small.

#### Ephemeris Errors

Milliken & Zoller chose to combine SV clock errors and ephemeris errors since the two are indistinguishable in the solution. Ephemeris errors arise from inaccuracies in the determination of the SV's location in space. This is calculated at the MCS after the four monitoring stations feed the location information to it. The four monitoring stations all simultaneously receive the signal from an SV and calculate its position using a technique analogous to that employed by a single receiver obtaining signals from four satellites. Such tracking occurs daily over a long period of time and the satellite ephemerides are continually refined. Ephemeris prediction parameters are then uploaded in the SV navigation message by the MCS. These parameters are used by

the receiver to calculate the position of the SV when the signal was transmitted. The error in this location, refinement, and prediction process, plus the residual clock error, is 1.5 meters.

#### Group Delay

Group delay is "the delay resulting from uncertainties caused by the processing and passage of the signal through the SV equipment." The delay is satellite specific and each satellite is tested and calibrated for its group delay before its launch. The delay is included in the clock correction parameters in the navigation message. Unresolved group delay results in an estimated error of one meter.

#### Receiver Noise and Resolution

Similar to the SV group delay but at the other end of the channel is the delay due to receiver noise and resolution. This is the unquantified delay as a result of processing the signal through the receiver hardware and manipulating it with software information. The error produced from these sources will be specific to each individual receiver. Milliken & Zoller estimated that a high-performance four channel receiver would yield a receiver noise and resolution error of about 1.52 meters. This is the amount allowed as a maximum in the GPS system specification document. In fact, the one sigma error is expected to be 2.6 meters for the C/A code and 0.4 meter for the P-code (TR 82-2, 1982:2.0-10).

To understand the derivation of these measures the carrier to noise density ratios must first be specified. The following discussion, through the pseudorange noise errors table, was extracted from TR 82-2

(1982:2.0-6 thru 2.0-7). The variance of the pseudorange error resulting from receiver noise is

$$\sigma_R^2 = (W^2 B_R) / (C/N_O)$$

where

$W$  = code chip width  $(1/1.023\text{Mhz}) \times (3 \times 10^8 \text{m/s}) = 293.3$  meters for the C/A code and  $(1/10.23\text{Mhz}) \times (3 \times 10^8 \text{m/s}) = 29.3$  meters for the P-code

$B_R$  = bandwidth of the receiver (assumed 1 Hz)

$C/N_O$  = the carrier to noise density ratio

$N_O$  is the receiver antenna output noise power/hertz,

$$N_O = k T_R$$

where

$k$  = Boltzmann constant,  $1.38 \times 10^{-23}$  joule/K

$T_R$  = effective receiver system noise temperature (assumed 580K)

So  $N_O$  is -201 dBW/Hz. Table III lists the required receive signal or carrier levels as per the GPS specification. The resulting carrier to noise density ratios ( $C/N_O$ ) are shown in Table IV. When these values are used with the variance of pseudorange error equation the RMS pseudorange noise errors can be found. These results are in Table V.

Table V reflects a realistic estimate of an actual value for receiver noise errors attainable and shows an improvement over the specification constraint shown in Tables I and II. This new, more realistic value will be entered in the GPS real time error budget, Table VII.

TABLE III

Received Carrier Signal Levels by Specification (TR 82-2, 1982:2.0-6)

Channel	C/A Code	P-Code
L1	-160 dBW	-163 dBW
L2	-166 dBW	-166 dBW

TABLE IV

Carrier to Noise Density Ratios,  $C/N_0$  (TR 82-2, 1982:2.0-6)

	C/A Code		P-Code	
	L1	L2	L1	L2
$C/N_0$ (dBHz)	41	35	38	35

TABLE V

Pseudorange Noise Errors (1 sigma) (meters)

	C/A Code		P-Code	
	L1	L2	L1	L2
$\sigma_R$ (meters)	2.614	5.216	0.369	0.522



### Atmospheric Delays

Delays of the signal through the atmosphere are often placed in two categories, ionospheric and tropospheric delays. Both delays occur because an electromagnetic wave travels more slowly in media such as the atmosphere than in the vacuum of space, and because the wave bends as it passes through media of changing density. The mathematical formula describing this is

$$v = c/n$$

where

$v$  = speed of the electromagnetic wave in the medium

$c$  = speed of the electromagnetic wave in a vacuum

$n$  = index of refraction, the positive square root of the product of the relative dielectric constant and the relative magnetic permeability of the medium

A physical model of the index of refraction can be given in terms of the number density of electrons in the path of the signal (Glasstone and Dolan, 1977:493).

$$n = [1 - (0.8)N/(10^4 f^2)]^{1/2}$$

where

$N$  = number density of electrons (electrons/cm<sup>3</sup>)

$f$  = frequency (MHz)

This equation gives insight to the mechanism by which the ionosphere causes signal delay. When the signal passes through the ionosphere it causes the free electrons to oscillate at the frequency of the signal. As the number of electrons decreases at a given frequency,  $n$  will increase and the velocity of the wave will decrease. This happens because when electrons oscillate they are undergoing acceleration and

therefore will emit an electromagnetic wave of the same frequency as their oscillation. This secondary electromagnetic wave, when superimposed on the initial wave, forms the "total wave" (Gartenhaus, 1977:22). It is this total wave that has the velocity described by  $v = c/n$ .

The delay caused by the formation of the total wave can be removed analytically. According to E. H. Martin of the Magnavox Government and Industrial Electronics Company (1980:115), the ionospheric delay can be expressed as

$$\Delta L = (-b/4 f^2) I_V [\csc (E^2 + 20.3^2)]^{\frac{1}{2}}$$

where

$\Delta L$  = ionospheric delay in meters

$b$  =  $1.6 \times 10^3$  (constant in MKS units)

$f$  = carrier frequency in hertz

$I_V$  = vertical electron content in electrons per meter

$E$  = elevation angle in degrees

Since the carrier frequencies are given for L1 and L2 in the GPS system the only variable in the equation is the electron content along the line of sight between the receiver and satellite. This is expressed in terms of the vertical electron content and the the elevation angle of the satellite (Martin, 1980:115). As the elevation angle decreases the electron content increases since the signal must travel through a thicker slice of the ionosphere. And, of course, as the electron content increases, the delay increases.

The electron density in the ionosphere is dynamic both spatially and temporally. Two key drivers are solar activity and the latitude of

the receiver. Increased solar activity will bring increased electron density. When the atmosphere is no longer irradiated by the sun the free electrons recombine with ionized gases and the gases are not reionized due to the lack of an energy source (solar rays) to power the process, thus there are lower electron densities at night. Unfortunately, it is not possible to know the electron number densities with sufficient accuracies at all times and places as would be required using the GPS system. Accordingly, an more practical method of removing ionospheric delay was developed.

This method removes ionospheric delay by using two frequencies (L1 and L2) to transmit the same message at the same time. The propagation times of the two signals can be compared and since the amount of refraction of the wave is frequency dependent, the delay can be very closely estimated and removed. As originally documented by Dr. R. A. Brooks in his Technical Note entitled Ionospheric Refraction Compensation in GPS Applications (1982), this process begins by finding the difference in the range measurements given by the two carrier frequencies L1 and L2,

$$\Delta R = R_2 - R_1$$

where

$R_1$  = pseudorange calculated using L1

$R_2$  = pseudorange calculated using L2

and calculating a constant as follows:

$$K = (f_2)^2 / [(f_1)^2 - (f_2)^2]$$

assuming the L1 frequency is to be corrected and  $f_1$  and  $f_2$  are the

respective link carrier frequencies. Since  $L_1$  is the frequency to be corrected

$$K = (1227.6 \times 10^6 \text{ Hz})^2 / [(1575.42 \times 10^6 \text{ Hz})^2 - (1227.6 \times 10^6 \text{ Hz})^2] = 1.55$$

Then simply multiply the range difference by the constant  $K$ . The product is the delay that should be subtracted from  $R_1$ .

Now since  $L_1$  and  $L_2$  were used together to obtain the pseudorange, the variance in the noise of the two channels must be considered together. This will lead to a one sigma UERE due to receiver noise when this ionospheric delay compensation is employed. The  $L_1$  (and  $L_2$ ) noise variance due to ionospheric delay is

$$\sigma_{D1}^2 = K^2(\sigma_{R1}^2 + \sigma_{R2}^2)$$

where  $\sigma_{R1}^2$  and  $\sigma_{R2}^2$  are the pseudorange noise errors found in Table V. The total residual measurement noise variance is

$$\sigma_{R+D}^2 = (1+K)^2 \sigma_{R1}^2 + K^2 \sigma_{R2}^2$$

Performing these calculations, the one sigma RMS noise errors with and without the ionospheric delay included are shown in Table VI (TR 82-2, 1982:2.0-7 thru 2.0-9).

The above description of ionospheric delay removal was, of course, for the case involving use of both link carrier frequencies. If the GPS system is being used with only one of the frequencies then a model of the ionosphere can be employed to estimate the delay. Such a model is dependent upon the same factors as the analytical expression. There are several small models available which may eventually be suitable for

TABLE VI

RMS Noise Errors Before and After Dual Frequency Ionospheric Delay  
Compensation (TR 82-2, 1982:2.0-9)

	C/A Code	P-Code
$\sigma_R$ (meters)	2.614	0.369 (before)
$\sigma_D$ (meters)	9.018	0.988
$\sigma_{R+D}$ (meters)	10.454	1.238 (after)

field use (Geckle and Feen, 1980; Klobuchar, 1982). In addition to these there is a large, comprehensive model developed and used by the Johns Hopkins University Applied Physics Laboratory. This model can be run for post-test processing of recorded GPS data.

Tropospheric delays are not frequency dependent so the use of two frequencies is of no consequence here. Instead, geometric models of the depth of the troposphere a signal must penetrate depending on its elevation angle from the receiver have been developed.

The changes in water content of the transmission media increase the relative dielectric constant of the air and is the cause for the refraction of the carrier. This is different from ionospheric delay which is caused by electron content of the media. Since the water content of the atmosphere increases as the signal travels from the upper atmosphere to the surface, the amount of refraction also changes with the speed of the carrier becoming slower as it nears the surface. This is dependent upon the same formula as the ionospheric delay, namely,  

$$v = c/n.$$

Using the troposphere model provided by E. E. Atkshuler and P. M. Kalaghan of the Air Force Cambridge Research Laboratories, Martin stated that a one sigma UERE of 0.1 meter remaining after tropospheric delay compensation would be a reasonable estimate. However, TR 82-2 uses a value of 2.0 meters for the estimate of tropospheric error (1982:2.0-10). This more conservative value will be used for this study. Note that more sophisticated models along with more information on local conditions affecting atmospheric water content could yield a greatly improved UERE. Currently, however, such methods are only available for post-test processing.

#### Multipath Errors

Multipath errors result from the recombination of signals at the receiver. These signals are actually the same signal emanating from the same satellite but they travelled over different routes between the satellite and the receiver. This can occur if the signal is reflected from surfaces near the receiver. The signal received directly from the SV and the one received after it reflects can interfere with one another resulting in an inaccurate representation of the original signal.

E. T. Fickas of SRI International documented testing for multipath over short path differences. The C/A code was able to track through multipath interference from nearby reflecting surfaces. The Analytic Sciences Corporation offered an explanation for this (TR 82-3, 1983:3-9). Once the receiver is locked on to the signal the second signal from the multipath appears to the receiver to be a signal from a "repeater jammer". The receiver can discern the difference between the direct signal and the multipath signal due to the steep autocorrelation peak of

the code. Since the second signal looks like noise to the receiver and the C/A code offers 61.8 dB Hz protection against noise, the second signal, the multipath signal, has little effect.

According to Fickas, however, additional path lengths of up to 1.5 chips can cause interference. A chip is the distance equivalent to the time required for one cycle of the appropriate code frequency to occur. For 1.5 chips this distance is 44.0 meters for the P-code and 440.0 meters for the C/A code (1983:199). The actual amount of multipath differential delay is about (TR 82-2, 1982:3-8)

$$\Delta T = (2h/c)\sin \theta$$

where

$h$  = user altitude

$c$  = speed of light

$\theta$  = elevation of the indirect signal path relative to the ground

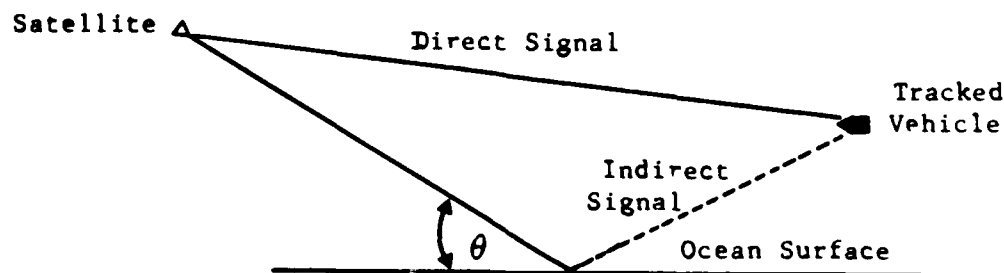


Fig 4. Multipath Geometry

Based on this relationship the maximum user height below which multipath becomes a factor using the C/A code is 220 meters if the elevation angle between the user and the satellite is 90 degrees. If the elevation angle is 5 degrees the maximum height is 2530 meters.

In addition to the magnitude of the multipath delay, the amount of multipath induced error is also dependent upon the ratio of the multipath signal to the desired signal amplitude. The larger the ratio, the larger the delay. At this time it is not possible to know relative signal amplitudes so the tracking error cannot be found exactly. Suffice it to say that the maximum user height below which multipath becomes a factor using the C/A code is well below some expected test flight altitudes. For the purposes of this study, a one sigma UERE of 12.0 meters due to multipath using the C/A code will be used. This value complements the that used for the P-code in the system specification (TR 82-2, 1982:2.0-5).

#### Receiver Vehicle Dynamics

Coupled with the receiver noise and resolution is the receiver vehicle dynamics since this is also specific to the receiver under particular circumstances. For this reason, Milliken & Zoller did not include vehicle dynamics in their estimation of the range error.

In fact, the receiver vehicle dynamics can result in a substantial error in the TSPI if not properly accounted for. The motion of the vehicle could block the receiver antenna for long periods. Should this occur the receiver must once again acquire the GPS signals. Outages could occur which are of sufficient length to lose significant quantities of information.

Another potential problem is the update times. For a system not using a translator (to be explained later) the TSPI solution is obtained at discrete intervals, possibly as small as one second. If the receiver moves a great deal between updates, it could introduce error into the



new solution simply because it is so far away from where it was when the solution was previously updated. Still another problem is that the high dynamics causes an artificial phase displacement of the incoming signal. This phase displacement could be so large as to cause the loss of lock on the signal so it must be reacquired. Several techniques exist to reduce or prevent these circumstances. These techniques will be discussed at length later.

#### Total Error

The total system ranging error is the root sum of squares (rss) of the individual contributors to the error budget. The result is a user equivalent range error of 5.3 meters according to the system specification. Improving on the information offered by the specification, a real time error budget was defined by Federal Electric (TR 82-2, 1982:2.0-14) and is shown in Table VII. The difference between Table VII and the system specification is that the values for ionospheric delay compensation and receiver noise from Table VI are used in the real time error budget.

Note that the ionospheric delay compensation and receiver noise figures have been updated with the values from Tables V and VI. While the amount of multipath error is highly variable depending on the range and test configuration, for this estimation it was estimated to be  $W/25$ , where  $W$  is the code chip width. This is 12.0 meters for the C/A code and 1.2 meters for the P-code. Another feature of Table VII is that both the C/A and P-codes are represented. So the final one sigma UERE for the P-code is 4.7 meters and 16.6 meters for the C/A code.

TABLE VII

GPS Real Time Error Budget (TR 82-2, 1982:2.0-10)

Error Sources	One sigma UERE (meters)	
	P-code	C/A code
SV clock and ephemeris errors	2.7	2.7
Group delay	1.0	1.0
Ephemeris prediction and model implementation	2.5	2.5
Tropospheric delay compensation	2.0	2.0
Ionospheric delay compensation	1.0	9.0
Receiver noise	0.4	2.6
	1.2      10.5	
Multipath	1.2	12.0
Other	0.866	0.866
RSS	4.7	16.6
RSS (with no ionospheric delay compensation)	4.6	13.0

The above figures are for the case involving compensation for the ionospheric delay error using both L1 and L2. If only one link is to be used, as is the case with missile tracking, the errors are somewhat different. In this situation the error due to ionospheric delay compensation must be removed from the table. This leaves a new one sigma system UERE of 4.6 meters for the P-code and 13.0 meters using the C/A code. While this shows an improvement in the UERE recall that the entire ionospheric delay error is now present in the solution but is not accounted for in the 4.6 and 13.0 meter figures.

Another source of error not considered in this table is vehicle dynamics. As will be further detailed in Chapter IV, if the receiver is

on a vehicle undergoing high acceleration the accuracy of the position and velocity solutions may be degraded significantly. It will be assumed this could be the case with some SRAM II test trajectories. Unfortunately, no analysis of the magnitude of error introduced by dynamics is available. Nevertheless, it is certain that significant degradation will occur and that this error contribution must be specifically addressed in the final equipment configuration.

Finally, note the figures shown in Table VII do not reflect the "accuracy" of the system. The accuracy of the TSPI is also dependent upon the relative positions of the four satellites used and the receiver. The measurement of this uncertainty can be accomplished using dilution of precision equations. These equations will be covered in Chapter IV.

#### IV. System Accuracy

The accuracy of the TSPI given in the GPS solution is actually the product of the ranging error, described above, and a factor called Geometric Dilution of Precision (GDOP). This factor is dependent upon the relative positions of the four satellites used in the solution and the receiver. There are four GDOP parameters (Jorgensen, 1984:1-12), the most often used of these is Precision Dilution of Precision (PDOP). Position dilution of precision is the rss of the three components of position error. The PDOP multiplied by the range error yields the radial error in user position in three dimensions. The average PDOP values fall between two and four, and obviously the smaller the value the greater the accuracy of the TSPI.

The other three parameters of GDOP are Horizontal Dilution of Precision (HDOP), Vertical Dilution of Precision (VDOP), and Time Dilution of Precision (TDOP) (Milliken & Zoller, 1980:10). HDOP is the uncertainty in the two horizontal dimensions. VDOP is the uncertainty in the vertical direction and TDOP is the uncertainty in time or user clock bias. Here again, the accuracy of the TSPI is given by the product of the particular GDOP parameter and the range error. Therefore, HDOP X Range Error is the radial error in the horizontal plane, VDOP X Range Error is the vertical error in position, and TDOP X Range Error is the range equivalent error of the user clock offset. As before, the smaller the GDOP parameter, the more accurate the respective measurement.

The GDOP value at a particular location will vary with time since the GDOP is dependent on the relative configuration of the four chosen satellites and the receiver. These satellites are moving in their orbits and the receiver is moving relative to the satellites due to earth rotation and movement of the receiver relative to the earth. At the same time, the GDOP in one location will be different from that of nearby locations due again to the change in relative positions. With these time and space dependencies for GDOP there is no analytical solution to indicate the correct statistical values of GDOP parameters. In addition, while normally there are four satellites in a configuration yielding a reasonable GDOP value, this is not always true. On those occasions when there are not four satellites in view or when they are all very close together, GDOP values of several hundred or a thousand are expected. Since there are usually more than four satellites in view at any one time, the system can escape from these very high values.

When the GPS receiver begins to solve for its TSPI it can choose any four of the satellites in its view. After it acquires the first satellite it will receive the almanac information detailing the location of the other satellites. The receiver can feed this information into an algorithm which will quickly solve for the best satellites to use yielding the minimum GDOP (Chen, 1984:332-338). The receiver can then acquire each of these satellites and go on to solve for TSPI.

Velocity measurements have associated errors which are dependent upon the same GDOP parameters as the time and position information. According to Milliken & Zoller (1980:12), accuracies of 0.06 to 0.15 m/s are expected from the GPS system. This assumes a receiver moving at constant velocity and averaging intervals of about one second.

MacDonald and Jones (IEEE National Telesystems Conference, 1983:192) were more conservative in their estimate of velocity accuracy. They calculated a velocity accuracy of .04 - .44 m/s at constant velocity, but with averaging intervals of 0.1 to 1 second.

The accuracy of the GPS system as described thus far sheds an unfavorable light on its use for SRAM II tracking. A 16.6 meter UERE that must be multiplied by a PDOP value of, say, two or three will certainly not be sufficiently accurate. There are, fortunately, many methods of enhancing GPS accuracy. Three of these are applicable to small missile tracking: the differential method, use of a high dynamic low volume (HDLV) receiver, and post-test processing.

Differential Method. The differential method removes the error caused by ionospheric delays (Fickas, 1983:195) as well as other bias errors. These errors are actually random errors but they change so slowly that they can be periodically measured and removed just as if they were biases to the pseudorange measurement. Recall that nearly all the ionospheric delay can be removed from the TSPI solution if both L1 and L2 are received and processed. For the SRAM II tracking application, however, only one link will be relayed by the translator (translation will be discussed in Chapter V). Therefore the errors due to ionospheric delay must be removed using another method. Currently the best alternative is the differential method.

The differential method consists of placing a receiver of known location along the test flight path. The receiver at this known location uses the GPS signals to solve for its position. It then

compares the GPS solution to the known location. The difference between the two is called the system bias. This bias can be immediately transmitted to the test flight vehicle and fed into its GPS processor. The test vehicle's processor will then use the bias information to reduce the error in its own solution for position.

A variation of this involves simply recording the differential receiver's raw signals as received, or recording its uncorrected TSPI solution. The recording is accomplished such that the information from the tracked vehicle and that from the differential receiver can be exactly correlated in time. Then the signals can be compared and the bias removed in post-test processing.

To get an idea of the error improvements possible with the differential technique consider tests done by SRI International involving GPS solutions for TSPI (Fickas, 1983:194-201). Table VIII shows a substantial improvement using the differential method. The absolute accuracy level of accuracy in these tests were not very good but this is a function of the particular equipment used. The important point to note is the difference between values for differential and non-differential applications.

Even with the greatly increased accuracies of the differential system there are some points of practical concern. These are geometric decorrelation and atmospheric uncertainties.

Geometric decorrelation refers to inaccuracies involving the satellite ephemeris. Recall the Control Segment monitors the position of the satellites and periodically updates the ephemerides stored aboard the satellites. These ephemerides are transmitted with the navigation

TABLE VIII

An Example of Actual Error Reduction Through the Use of the Differential  
Method (Fickas, 1983:199)

Receiver Separation (nautical miles)	Combined Bias and Noise Error (meters)	
	Differential	Non-differential
0.05	8.1	18.9
	5.9	11.2
25	9.4	15.6
135	10.0	17.24
	7.4	17.2
150	8.8	12.5
240	4.3	12.9
	9.5	11.5
Mean (std dev)	7.93 (1.96)	14.64 (2.97)

message and are an integral part of the TSPI solution. Any error in the ephemerides due to unmodelled drift, for example, will produce an error in the position solution. Even though the differential technique removes most of this error, the physical separation between the tracked vehicle and the differential receiver makes it impossible to remove all the error. It is not possible to quantify the error beforehand without knowing the direction of the initial ephemeris error. However, the error is maximized for displacements which are first, orthogonal to the bisector of the angle with the satellite at the vertex and the receive stations at the legs, and second, in the plane defined by the satellite,



the differential station and the tracked vehicle. In this situation the maximum error is approximately

$$UERE = de/h$$

where

d = separation distance between the two receivers

e = ephemeris error

h = satellite altitude

This error is about 2.4 meters for an ephemeris error of 500 meters and a tracked vehicle to master receive station separation of 100 km according to Kalafus, et al (1984:206).

Incidentally, 100 km is assumed here to be the approximate limit of the range for telemetry gathering. This is based on an elevation angle of five degrees which corresponds to the mask angle of the GPS receivers. The mask angle is the minimum angle up from the horizon for which a satellite is in view. It is a function of the user's antenna elevation angle. As the angle is decreased the signal must travel through a thicker cut of the ionosphere and troposphere and so more uncertainty is introduced into the pseudorange measurement. Therefore, there is a tradeoff between lowering the mask angle to allow as many satellites as possible to be in view and raising it to allow only the most accurate pseudorange measurements to be taken. The optimum is currently considered to be five degrees (Spilker, 1980: 33).

The tracked vehicle must remain in view of the master receive station if it is to be able to transmit data that can be successfully received by the master receive station. It must therefore remain above the limit imposed by the five degree mask angle. The ground range limit

can be found as a function of the tracked vehicle altitude (LRPS 86-5, 1986:21).

$$G_R = 60(90 - [E + \sin^{-1}(R/(R + h))\cos E])$$

where

$G_R$  = ground range

$E$  = mask angle

$R$  = earth radius (3440 nm)

$h$  = altitude of the tracked vehicle above the surface of the earth

A list of the ground ranges as a function of tracked vehicle altitude is given in Table IX.

The atmospheric uncertainties cause errors which cannot be entirely removed from the TSPI solution. These errors have two main sources: ionospheric and tropospheric, just as in the normal GPS error budget. The ionospheric error that remains after differential corrections has

TABLE IX

Maximum Ground Range vs Altitude, 5 Degree Elevation Angle

Altitude (meters)	Altitude (nm)	Ground Range (km)	Ground Range (nm)
100	0.05	1.05	0.57
500	0.27	5.69	3.07
1000	0.54	11.32	6.11
2000	1.08	22.39	12.09
3000	1.62	33.26	17.96
4000	2.16	43.93	23.72
5000	2.70	54.41	29.38
6000	3.24	64.71	34.94
7000	3.78	74.84	40.41
8000	4.32	84.82	45.80
9000	4.86	94.64	51.10
10000	5.40	104.30	56.32

two basic sources. The first of these is the different path lengths through the ionosphere a signal must traverse coming to the differential receiver and the tracked vehicle from the same satellite. The different path will result in a slightly different delay that cannot be removed. The error changes with distance between the differential receiver and the tracked vehicle but at a separation distance of 50 km the error is less than one meter (Kalafus, et al, 1984:207).

The second source of error is differences in the ionosphere itself. Irregularities in the ionosphere can be fairly localized. An irregularity may therefore not be shared by two nearby signal paths. In such instances the differential method cannot remove the error. This accounts for errors on the order of one-half meter (Kalafus, et al, 1984:207).

Errors remaining in the solution and emanating from the troposphere are essentially like those of the ionosphere. They result from different path lengths and variances in the gaseous medium along the two paths. Such variances bring changes in the atmosphere's dielectric constant as detailed earlier. The end result is a difference in the amount of time required for the signal to propagate over the same path length, yielding a different pseudorange measurement. In addition, the dielectric constant can undergo dramatic changes below about 3000 meters (10,000 feet). Since the differential receiver will be at sea level and the SRAM II will be at some higher altitude, this can be expected to introduce rather large errors. If the elevation angle to the satellite from the tracked vehicle is five degrees, this error will be about 1.2 meters (Kalafus, et al, 1984:206-208).

Another error source is the "differential station uncertainty" according to Kalafus. This is due to noise and results in an error of 2.2 meters. User receiver noise, on the other hand, is an even worse problem. It results in an error of 7.5 meters (one sigma). When all these error sources are taken into account a total one sigma error (rss of the above sources) of 8 meters is apparent. With heavy post-test processing the error can be smoothed to four meters according to Kalafus.

It should be noted that his calculations are based on a sequential, or single channel, receiver. Such a receiver can process signals from only one satellite at a time. This technique leads to large errors in dynamic situations. The figures quoted above are for an airborne receiver so the coupled effects of the sequential receiver and the dynamic environment resulted in unusually large errors.

A multi-channel receiver, on the other hand, can receive signals from four or even five satellites simultaneously. The advantages of such a receiver were summarized in an IEEE paper written by Ashjaee and Helkey (1984:242). They are, (1) no need to reacquire satellites since four satellites (or five, as the case may be) are tracked simultaneously, thus saving time otherwise required for acquisition, (2) TSPI is obtained and updated faster, and (3) satellite relative positions that can lead to serious decreases in accuracies can be detected and avoided.

Another consideration was drawn from an article by Russell and Schaibly (1980:80) specifically dealing with ephemeris errors. They predict an error of only about 15.0 meters in the direction of satellite travel and even smaller in all other directions. Adopting this 15.0

meter ephemeris error and using a separation distance of 100 km and a satellite altitude of 20,183 km, the UERE due to geometric decorrelation is only about 0.1 meter. With a 50 km separation the error drops to about 0.04 meter which is negligible.

Therefore, while Kalafus' discussion of differential operation brought out the important concepts of geometric decorrelation and atmospheric uncertainties, the actual errors he used are too pessimistic.

High Dynamic Low Volume Receivers. The second technique to improve GPS accuracy is use of high dynamic receivers developed by the Jet Propulsion Laboratory of California Institute of Technology (Hurd, 1983). This technique uses maximum likelihood estimators to estimate the position and velocity of the platform. Recall that the GPS receiver finds the pseudorange from the receiver to a satellite by comparing the phase of the incoming signal against a self-generated signal. The phase difference indicates the amount of time required for the signal to travel from the satellite to the receiver (Arnold, 1983:234). However, in a high dynamic environment (high acceleration turns) like that of a tactical missile, the acceleration of the missile on which the GPS receiver is riding can be so great that the phase is displaced even further. If this displacement is too large the signal processor cannot measure the phase change and therefore cannot solve for pseudorange.

The JPL high dynamic receiver works in a unique fashion to avoid this problem. Instead of attempting to directly compare the incoming and the self-generated signals for phase displacement, the HDLV multiplies the incoming signal by each possible value for delay in units

of time. It then measures the signal energy at each of these products (signal times delay) at every frequency. The maximum likelihood estimate is the value of time delay and frequency with the highest energy (Hurd, 1983:221). This information is then used to find the phase displacement between the incoming and self-generated signals even if the displacement is large. The processor can then solve for position and velocity. The report issued by Cal Tech's Jet Propulsion Laboratory indicates a one sigma rms accuracy of .4 meters with a S/N of 38 dB-Hz under a 50g circular acceleration and 40 g/s jerk (Hurd, et al, 1985:5-16). This accuracy was obtained using the P-code under conditions simulated in the laboratory and is the UERE (user equivalent range error) since it has not yet been multiplied by a PDOP value. Although Hurd, et al, did not test the procedure using the C/A code they claim errors as low as five meters can be obtained in this environment. This figure is obtained for the case of absolute navigation. If differential navigation were used, here again the accuracies would be greatly increased.

Post-Test Processing. The post-test processing mentioned above is an extremely effective means of improving overall accuracy. It consists of first recording the signals from the GPS receive equipment. Then, free of constraints imposed by real time processing, the signals can be repeatedly smoothed and filtered until nearly all the bias error and most of the random (noise) error has been removed. Using this post-test processing should reduce the C/A code one sigma UERE from 13.0 meters to 1.7 meters according to the study documented in TR 82-2 (1982:2.0-11 and 2.0-15), as shown in Table X.

TABLE X

TR 82-2 Post-Test Error Budget (1982:2.0-14)

Error Source	User Equivalent Range Error (1 sigma) meters
Clock, navigation subsystem, SV perturbations, and ephemeris smoothing	1.5
Tropospheric delay compensation	0.2
Ionospheric delay compensation	0.45
Receiver noise	0.13
Multipath	0.6
Other	0.5
RSS	1.7

Furthermore, a good approximation of the UERE using the differential method with post-test processing can be obtained. This is done by removing the bias errors from the error contributors. This leaves only multipath ( $0.6 \sqrt{2}$  meters) and receiver noise ( $0.13 \sqrt{2}$  meters). An additional error factor, receiver bias, must be considered in the differential case to account for differences between the two receivers so a one sigma UERE of  $0.5 \sqrt{2}$  meters will be included. The multiplication by root two is due simply to the fact that the error occurs both at the differential receiver and the tracked vehicle. The rss of the two errors is the UERE at one receiver multiplied by root two. The one sigma UERE for use of the C/A code with the differential technique and post-test processing is 1.1 meters.

Unfortunately, the TR 82-2 study did not include geometric decorrelation and atmospheric uncertainties as introduced by Kalafus, et al (1984:206). Recall that the Kalafus article specified an error due to geometric decorrelation which turned out to be negligible when an ephemeris error of 15.0 meters was used. In addition, the article introduced the difference in path lengths through the ionosphere resulting in 0.5 meter errors, and differences in the ionosphere itself between paths resulting in errors of 0.5 meters, and finally an error due to tropospheric bias of 1.2 meters.

There is, however, a problem with translating Kalafus' figures to the TR82-2 figures. Recall that Kalafus, et al used a sequential receiver operating on the P-code, and a post-test accuracy improvement of 50% (from 8 meters to 4 meters). The requirement here is to use a four channel receiver with the C/A code and to specify the post-test processing improvements for each individual error.

To adjust for these differences first note that errors will increase when moving from the P to the C/A code. On the other hand, the errors could decrease when going from a sequential to a four channel receiver if there are significant accelerations involved. But Kalafus' figures for geometric decorrelation and atmospheric uncertainties are essentially the same for the faster airborne and slower marine/land receivers (1984:206-207). This indicates no difference in accuracy between a sequential receiver and that of a four channel receiver for these particular errors at the level of dynamics presented by the airborne platform. Based on this it will be assumed that geometric decorrelation for a C/A code four channel receiver is 0.5 meter, and atmospheric uncertainties total to 5.0 meters before post-test



processing. After post-test processing the geometric decorrelation error is assumed to be zero and atmospheric uncertainties total to 0.5 meters. These figures are considered to be very conservative. Given this, the post-test error budget for differential operation using the C/A code can be estimated. The new one sigma UERE is 1.2 meters. This new UERE is obtained by taking the rss of the multipath, receiver noise, and receiver bias figures used earlier, and including the 0.5 meter error for atmospheric uncertainties.

## V. Translation

For the particular problem of using GPS to track SRAM IIs during test flights the system can be modified to include the use of a translator on board the missile instead of a GPS receiver. The translator simply gathers the signals from all available GPS satellites and retransmits them to a master receive station. This method is not limited to using only four acquired satellites. Instead, all GPS satellites in line of sight from the translator can be recorded.

The motivation for using a translator is that the SRAM will be destroyed at the conclusion of its flight. Fully equipped GPS processors are relatively expensive and as yet are heavy, large and have high processor power requirements (see Table XI). On the other hand, disadvantages sustained by using translators are a decrease in tracking accuracy, an increase in RF bandwidth, and the power required for the transmitter. Overall, it is more advantageous to use a translator system than a pure onboard receiver relay (standard GPS processor).

Three translator systems are envisioned. These are the bent pipe linear relay, the transdigitizer relay, and nonlinear relays as listed in Table XII. The bent pipe system (TR 82-2, 1982:4.7-11) is used on the SATRACK system for Trident missile testing. Considerable experience and expertise exists for this system. It operates by retransmitting in real time all the GPS satellite signals received at the test vehicle during its flight.

TABLE XI

Comparison of Relay and Receiver Systems (TR 82-2, 1982:4.7-10)

PARAMETER	RECEIVER SYSTEM (HDUE)*	RELAY SYSTEM
Cost	high (25K)	low (5K)
Weight (lbs)	77	<5
Power (watts)	>200	<50 (Depends on relay transmitter class and power)
Size (cu ft)	3.5	<0.5
Time to first fix	152	152 (A rough position fix may be derived from less than 1 sec of continuous data through computer analysis of the relayed signals)
Code Demodulation	P, C/A	C/A
Pseudorange Accuracy	1.5 meters	10 meters
MTBF (hrs)	500	>2000
Loop Tracking Aid	Inertial aids required	No aid required for post operative data reduction. Aids required for near real time. May be derived from telemetry, radar or Doppler data.
RF Bandwidth	2 kHz or less depending on update rate	2 MHz (C/A code)
Transmitter Power	Very low	Up to 20 watts depending on range
Probability of acquisition	0.95	>0.95 if data reduction is post flight

\*HDUE is High Dynamic User Equipment

TABLE XII

Comparison of Relay System Characteristics (TR 82-2, 1982:4.7-20)

PARAMETER	BENT PIPE	TRANSDIGITIZER	NONLINEAR RELAY
RF bandwidth required	2.1 MHz (C/A) 10.1 MHz (P)	1.1 (C/A) 5.1 (P) (QPSK modulation)	2.1 MHz (C/A) 10.1 MHz (P)
Type of relay transmitter	Linear	Class C	Class C
Remodulation loss	0	7 dB	1 to 3 dB
Battery power drain	High (Due to class A final)	Low	Low
Compatible with No existing range receive/record equipment		Yes P code - no (bandwidth limit)	C/A code - yes P code - no (bandwidth limit)
Compatible with Yes standard GPS receiver		Yes	Yes
Susceptibility to jamming on relay link	Low (same as down link)	High	Low (same as down link)
Susceptibility to interception	High	Low if encrypted	High

In the case of the bent pipe system the idea is to relay the signal with as little change to its characteristics as possible. It is necessary to relay it at a different frequency than that used by the

satellites to prevent interference and feedback so the relayed signal is slipped to S-band. The transmitter must give the signal (with its embedded noise) enough power to allow the signal plus noise-to-noise ratio at the receiving ground equipment to be about the same as that of the incoming satellite signals to the translator. The translator must insure the amplitude and phase linearity of the signal is also unchanged.

While the above is an analog system, the transdigitizer relay system is digital. The system as outlined in TR 82-2 (1982:4.7-14) involves a down conversion of the incoming GPS signals to a 30 kHz nominal IF. This signal is hard limited. Limiting has the equivalent effect of reducing the S/N of the signal. The signal is then sampled for digitization but this again decreases S/N. The sampling rate is 2 MHz and results in a PCM-NRZ signal. Despite the reduced S/N this technique does offer the advantages of lower power requirements and the possibility for encryption of the signal prior to transmission to the ground facility.

In the third translator system, the nonlinear relay system, the incoming GPS signals are again down converted to the relay transmission frequency (S-band), then they are limited and filtered. The limiting reduces the S/N by about 1 dB. Finally, the signal is amplified and fed to a class C amplifier. This system requires a transmission bandwidth of about 2 MHz for the C/A code. It is possible to reduce the retransmitted signal bandwidth by down converting the incoming GPS signals to 30 kHz, limiting, filtering and then transmitting at the S-band frequency. The final bandwidth is about 1.1 MHz but this is at the expense of at least a 3 dB S/N loss. The advantage of the nonlinear

relay system is its lower power requirements, but there is a concurrent signal-to-noise ratio reduction.

When calculations for the pseudorange solution given by the GPS receiver were detailed earlier, the receiver was shown to be at the intersection of the surfaces of four spheres whose radii were given by the pseudorange from the satellite to the receiver. In the case where a translator is used the geometric visualization simply moves from spheres to ellipses (Wells, 1983:261) as shown in Fig 5. The vehicle is now located at the intersection of the ellipses. The ellipses share the master receive station as one focus and the respective satellites serve as the other focus for each ellipse.

The equation for the translated pseudorange sum measurement is

$$R_i = R_{STi} + R_{TM} + C(t_d + T_U - T_{Si})$$

where

$R_i$  = translated pseudorange measurement from the  $i^{th}$  satellite

$C$  = speed of light

$t_d$  = equipment delays and other errors

$T_U$  = user clock bias

$T_{Si}$  =  $i^{th}$  satellite clock bias

$R_{STi}$  = true range,  $i^{th}$  satellite to translator

$R_{TM}$  = true range, translator to master station

The range rate sum, to calculate the velocity of the vehicle, is unfortunately not nearly as simple. There are two reasons for this (Wells, 1983:261). The first is the remodulation of the signal. This is accomplished with the aid of an oscillator in the translator. Any error in the accuracy of this oscillator will be introduced into the

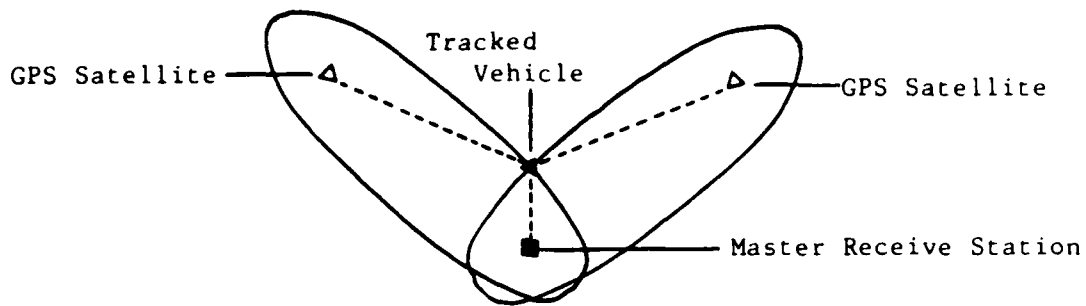


Fig 5. Translation Ellipses

signal and ultimately into the pseudorange rate measurement. Since the purpose of the translator is to reduce the loss of equipment at the culmination of the flight test, a highly accurate and expensive time standard such as a rubidium or cesium clock is not feasible. Instead a crystal oscillator must be used in spite of its inherent relative inaccuracy.

The second source of error due to translation is the additional doppler shift of the signal between the translator and the master receive station. This doppler shift will detract from the accuracy of the vehicle velocity measurement by incorrectly adding or subtracting a component of motion in the line of sight of the satellites.

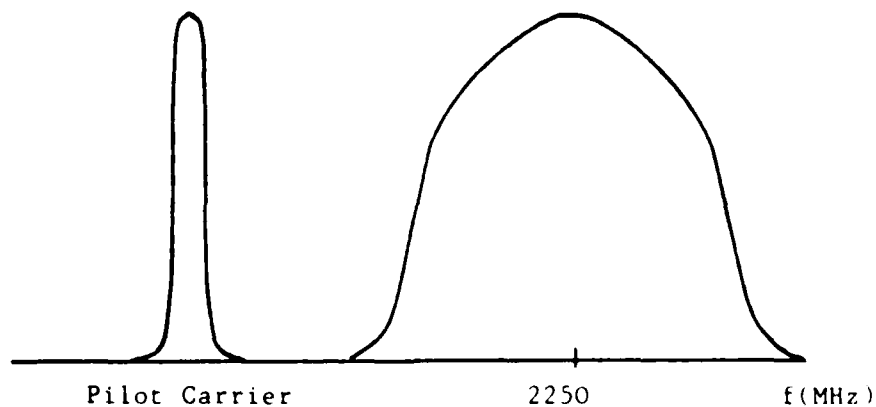


Fig 6. Translated GPS Spectrum

Fortunately there is a method to resolve these new problems and restore the accuracy of the system. When the signal is transmitted from the translator it is given a pilot carrier frequency in close proximity to the translated GPS spectrum as illustrated in Fig 6. Since the frequency at which the pilot carrier was transmitted originally is known, any deviation from this standard is due to doppler shift caused by motion of the vehicle along the line of sight to the master receive station. The known deviation can be applied as a correction to the translated GPS spectrum. This restores the signals to that of just after input at the vehicle receiver.

This explanation can be restated in mathematical terms. The following relationships were documented by Lawrence Wells of Interstate Electronics Corporation (1983:261-263). First, to find the frequency of a satellite carrier after translation

$$f_{Ti} = L1 (1 + \dot{R}_{STi}/C) + KLO (1\text{MHz} + e) \quad (2)$$

where

$f_{Ti}$  =  $i^{\text{th}}$  satellite translated carrier frequency

$L1$  = link 1 GPS satellite carrier frequency (1575.42 MHz)

$\dot{R}_{STi}$  = range rate from the  $i^{\text{th}}$  satellite to the translator

$KLO$  = translator local oscillator multiplier from a 1 MHz oscillator

$C$  = speed of light

$e$  = translator reference oscillator error

Fig 7 is a representation of the translator depicting where these changes to  $L1$  occur. Note the first term on the right hand side of Equation (2) is the  $L1$  frequency received at the translator and includes the doppler



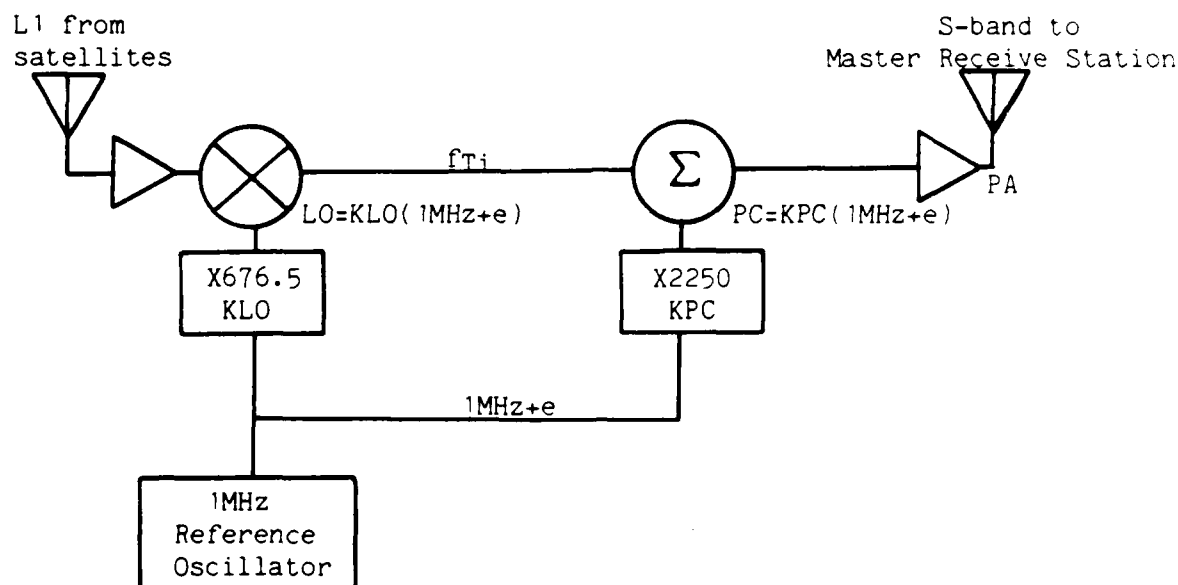


Fig 7. GPS Translator (Wells, 1983:262)

shift. The previous equation can be rewritten to a more understandable form.

$$f_{Ti} = L1 + KLO + KLO(e) + (L1)\dot{R}_{STi}/C$$

The first two terms to the right of the equal symbol constitute the nominal center frequency. The third term is the local oscillator error and the fourth term is the satellite to translator doppler shift term. The signal will now be transmitted through the channel from the translator to the master receive station and will be doppler shifted as noted above. The problem is to remove the oscillator error and the extra doppler shift introduced by the movement of the vehicle carrying the translator. Once this is successfully done, the signal will appear as it did when it was received by the translator. It can then be used to gain TSPI on the tracked vehicle.

To begin, note the pilot carrier and translated GPS spectrum can be treated separately.

$$f_{PC} = KPC (1 \text{ MHz} + e) (1 + \dot{R}_{TM}/C)$$

$$f_M = [L1 (1 + \dot{R}_{STi}/C) + KLO (1 \text{ MHz} + e)] (1 + \dot{R}_{TM}/C)$$

where

$f_{PC}$  = pilot carrier frequency

KPC = pilot carrier multiplier from a 1 MHz reference

$f_M$  = translated satellite carrier frequency at the master receive station

$\dot{R}_{TM}$  = true range rate from translator to master receive station

This shows that  $f_M$  is simply the translated carrier frequency described above corrected for doppler shifting occurring between the translator and the master receive station.

Now the pilot carrier frequency is known with the exception of the oscillator error. The oscillator error can be removed and the pilot carrier frequency shift can be used to backtrack and find the true carrier from the GPS satellite ( $f_{CARi}$ ).

$$f_{CARi} = f_M - [KLO/KPC] (f_{PC})$$

Substituting the equations for  $f_{PC}$  and  $f_M$

$$f_{CARi} = [(L1 (1 + \dot{R}_{STi}/C) + KLO (1 \text{ MHz} + e)) (1 + \dot{R}_{TM}/C) - KLO (1 \text{ MHz} + e) (1 + \dot{R}_{TM}/C)]$$

$$f_{CARi} = L1 (1 + \dot{R}_{STi}/C) (1 + \dot{R}_{TM}/C)$$

$$f_{CARi} = L1 + L1/C (\dot{R}_{STi} + \dot{R}_{TM} + \dot{R}_{STi}\dot{R}_{TM}/C)$$

At this point the signal is composed primarily of the L1 frequency and there are no remaining KLO or KPC terms. In addition, the oscillator error term has been removed so the frequency changes introduced during the translation process have been eliminated and the original frequency reconstructed. The exception is the remaining doppler adjustment for line of sight velocities between the translator and the master receive station. This term is still present and is not discernible from the satellite to translator doppler.

A serious problem with GPS translation remains unsolved, however. Since the carrier tracking loop must track the frequency through a wide range of frequencies due to these doppler shifts, the added translator to master receive station doppler can only make the task more difficult and sometimes impossible. The following is a solution to reduce the influence of this doppler term on the tracked frequency. After the signal is processed by the carrier tracking loop the appropriate terms are added back in.

This method consists of subtracting the entire pilot carrier frequency component from the translated signal.

$$f_{CARI} = f_M - f_{PC}$$

Making the same basic substitutions used in the last derivation yields

$$f_{CARI} = [L1 (1 + \dot{R}_{ST1}/C) + KLO (1 \text{ MHz} + e)] (1 + \dot{R}_{TM}/C) - KPC (1 \text{ MHz} + e) (1 + \dot{R}_{TM}/C)$$

$$f_{CARI} = [L1 (1 + \dot{R}_{TM}/C) + (KLO - KPC) (1 \text{ MHz} + e)] (1 + \dot{R}_{TM}/C)$$

At this point the signal is submitted to the carrier loop. Since most of the translator to master receive station doppler has been removed the carrier tracking loop must now sustain a dynamic environment roughly equal to that of a processor on board the missile itself. It can now acquire and track the GPS spectrum. To get back to a form that can be used for range and range rate measurements  $f_{PC}$  is scaled by  $(1-KLO/KPC)$  and added back in.

$$1 - KLO/KPC = (KPC - KLO)/KPC$$

$$f_{RRSi} = f_{CARi} + [(KPC - KLO)/KPC] f_{PC}$$

Making the familiar substitutions

$$f_{RRSi} = [L1 (1 + \dot{R}_{STi}/C) + (KPC - KLO) (1 \text{ MHz} + e)] (1 + \dot{R}_{TM}/C) + (KPC - KLO) (1 \text{ MHz} + e) (1 + \dot{R}_{TM}/C)$$

This is then submitted for range and range rate processing. Note this is the same relationship found in the first method. If the algebra is carried out it becomes

$$f_{RRSi} = L1 (1 + \dot{R}_{STi}/C) (1 + \dot{R}_{TM}/C)$$

which is exactly what was found in the first technique.

Since the last major component of range equipment has been introduced with the discussion of translators, it would be informative to summarize the range equipment configuration necessary for tracking SRAM test flights. The configuration begins with a multi-channel GPS receiver at the master receive station which is actively processing signals from the satellites. The receiver allows removal of all bias errors from the GPS solution since it is located at a precisely surveyed

point. Co-located with it are a recorder and telemetry gathering equipment to receive and record signals from the tracked vehicle. The vehicle, a SRAM II, is equipped with a translator which gathers signals from the satellites and retransmits them to the master receive station. There, they are recorded along with the signals going to the differential receiver.

This recording is then subjected to post-test processing. It is also possible not only to record the signals, but to process them in real time for TSPI. This involves two sets of signals, one from the differential receiver, one from the translator. As the solutions are calculated they can be filtered and smoothed, a process which removes many transient errors and results in an error reduction factor of at least 1/3 (Teasley, et al:1980:9). In addition, if the tracked vehicle is undergoing high dynamics, the JPL HDLV can be used to retrieve the accuracy lost due to accelerations. Therefore, two sets of data are generated, that which was recorded for post-test processing and the real time data for on-site tracking and monitoring.

## VI. Analysis

Finally, the task of determining whether or not GPS can be used to effectively track SRAM II test flights at ETR can be accomplished. This final analysis consists of measuring GPS accuracy against required standards.

### Standards

The standards the system must meet are contained in OR 1508. This operational requirement is generated for each test launch and the agency with responsibility for the missile sets the required standards. Strategic Air Command has responsibility for the SRAM missile. The standards used here will be the same as those used for the SRAM A follow-on test and evaluations (FOT&E). The requirements are for a position error of not more than 6.1 meters in each axis and a velocity error of not more than 15.2 meters/second in each axis at five points in the mission which are considered critical. These are the final aircraft checkpoint, the inertial guidance tangent plane initialization, missile launch, first pulse ignition, and fuzing. All other metric data shall be the best obtainable under existing conditions.

Put in other terms, for a live test run from eight minutes to four minutes prior to launch the accuracy must be the best obtainable. From four minutes prior to launch until loss of signal from the SRAM II or test termination by the Test Conductor, at least 6.1 meter position accuracy and 15.2 meters/second velocity accuracy are required.

## PDOP

The accuracy of the system is, of course, the one sigma UERE multiplied by the PDOP. Recall that the PDOP or Position Dilution of Precision varies across both space and time. The problem here is to find a representative value for PDOP that can be used in the final accuracy calculations. The appropriate estimator for PDOP depends not only on the location of the test range but also on the length of the test, and the time the test will be performed.

The PDOP values themselves are relatively easy to acquire thanks to a computer program supplied by Air Force Space Command. This program is government deliverable from a Scientific Engineering Technical Assistance support contract for Headquarters Air Force Space Command XPS. However, even though the PDOPs are obtainable there are many unknowns to deal with. As yet no range has been decided upon and of course the tests are yet to be scheduled so times are unknown. Nevertheless, a representative or at least reasonable range can be defined.

Based upon past usage and in-place telemetry gathering facilities ESMC has three choices for range location. These are near Antigua, Roosevelt Roads, and Bermuda. There are potential political problems associated with the use of Bermuda. Bermuda was eliminated as a possible site for SRAM A tests due to the sensitive nature of the telemetered data. It is expected that the same problems will hold for SRAM II testing.

Another consideration is the termination point of the test. It is required that the SRAM fall into the ocean where its eventual depth will prohibit recovery. This ensures the design of the missile remains

secure while foregoing the expense required to destroy or recover it. The two remaining sites, Antigua and Roosevelt Roads, both have adequate access to the Puerto Rico Trench. The Puerto Rico Trench reaches depths in excess of 6000 meters and is sufficient for securing the spent missile. The Roosevelt Roads site, however, is on the south side of Puerto Rico whereas the Puerto Rico Trench is to the north. The translated GPS signals must then pass above the raised landmass of Puerto Rico to reach the master receive station antenna. This could dictate a high elevation angle between the receive station horizontal and incoming signals. If this were the case, the SRAM tests would have to be performed at higher altitudes, possibly higher than desired for some FOT&E missions.

The minimum distance between the actual range and the data gathering stations would be about the same for both Roosevelt Roads and Antigua. On the other hand the Roosevelt Roads site is considerably closer to the Puerto Rico Trench. In fact, maximum distances between the test vehicle and the data gathering sites are as much as 50 - 60 kilometers less at Roosevelt Roads.

Nevertheless, due to the land obstruction of the Roosevelt Roads site the Antigua site was used for this study. The range was made to be equal in size to the range used for SRAM A tests at ETR although it is recognized that a larger or smaller range could eventually be used. The test range used here is rectangular with opposing corners at coordinates 16 degrees 40 minutes North, 61 degrees West and 19 degrees 40 minutes North, 59 degrees 30 minutes West.



The next step is to find how to get a representative PDOP value for this range recognizing that it may be necessary to use more than one value for different parts of the range at the same time. This required finding the temporal and spatial behavior of PDOP within the range.

First, PDOPs were calculated at specific time intervals for pairs of points in the range. Even when the two points were a full three degrees (about 335 km) apart the PDOPs in both locations at the same time did not vary significantly. In addition, the difference between PDOPs at 500 and 10,000 meters altitude is negligible. Therefore, it is only necessary to use one point in the range to represent the entire range. The location used was 17 degrees 38 minutes North, 61 degrees West. This is 80 kilometers northeast of Antigua. This is not the center of the range but was used since it is close to Antigua, as all launch points during tests are expected to be.

Second, the time variation of PDOP values was checked by calculating the values at the test point over time. It is only necessary to check the values over a 12 hour interval since the period of the GPS constellation is just under 12 hours. Calculating PDOPs at five minute intervals for 12 hours from 10:00 a.m. on 1 June 1987 yielded a range from 2.04 to 3.98. The data are listed in the Appendix.

It is customary to designate some percentile value of the PDOPs obtained as the actual PDOP estimator. This guards against the occasional unusually high values or "spikes" that can occur when the four satellites used for the solution approach a near planar configuration or are relatively close together. In these situations the PDOP can go very high, or be infinite (no value). If such values were used in a calculation of the mean it would be unrealistically skewed

upward. For the data from the test point and time interval detailed above, the 90th percentile value of the PDOP was 3.38. Roughly speaking, using this value implies that about 90 percent of the time the PDOP at this and nearby points will be 3.38 or better (less). About ten percent of the time it could be worse, but probably only up to 3.98, or .6 higher than the 90th percentile value.

While PDOP values vary significantly over time, there are short periods when they change very little. For example, calculating PDOPs at one minute intervals yielded periods where the PDOP changed only by .08 over the course of 15 minutes. The test planner can easily find such periods and schedule the test during a window when the PDOP changes little and is at a low value, say 2.2. This would yield a significantly better accuracy than the 3.38 90th percentile.

Therefore, while the majority of the literature uses a percentile for a representative value of PDOP, it is not necessary here. The percentile form is required for a generalized or global PDOP value, or for one where a "spike" is known to occur. Since the location of the range or at least a representative range is known here, there is no need to generalize the location. Furthermore since the period of the constellation is about 12 hours the behavior of the PDOP over the course of the 12 hours at some location can be tracked. Therefore, the lack of existence of PDOP "spikes" can be confirmed. For these reasons it is possible to use a PDOP value that is more representative of this particular case than the 90th percentile value. For these reasons, a realistic PDOP value of 2.2 will be adopted for the Antigua range.

In addition to the Antigua range, PDOP values were found for other locations within the ETR and for a point over the ocean at the Western Test Range near Vandenberg AFB, California. The first of these is the Ramey AFB Air Weather Station on the northwest coast of Puerto Rico. This site is not currently configured with telemetry gathering equipment but its location and proximity to the Puerto Rico Trench make it a possible alternative to Antigua. The data are shown in the Appendix. The PDOPs at Ramey are comparable to those at Antigua with a range of 2.05 to 4.14 and there is a 45 minute period with values of about 2.1 or less. The 90th percentile value is 3.5.

PDOPs were also evaluated at Roosevelt Roads and Bermuda. The Roosevelt Roads Test Point is 18.8 degrees North 65.7 degrees West and 500 meters altitude. PDOPs there range from 2.1 to 4.7 and have a particularly long low PDOP window of less than 2.2 for over 90 minutes. The 90th percentile value is 3.6. The Bermuda test point is 32.4 degrees North 64.6 degrees West and 500 meters altitude. The PDOP range is 1.9 to 256.5 with a 40 minute window of 2.2 or less and a 90th percentile value of 3.4. For the Western Test Range at Vandenberg AFB, California the test point was 34.0 degrees North and 12.2 degrees West and 500 meters altitude. PDOPs ranged from 2.0 to 11.8 with a 90th percentile value of 3.26. This demonstrates the flexibility of the system, and the ability to move the range to new locations should the need arise.

#### Test Accuracy

The particular nature of the GPS system and the test requirements allow breaking test accuracy down into two cases. The first case

involves the accuracy needed for range safety purposes. The second case is the tracking accuracy required for effective testing as defined by OR 1508.

For the first case, range safety, there are two scenarios. It has not yet been decided whether or not the test SRAMs will carry a command destruct package. If they do not then range safety personnel will not require any tracking data after the missile is launched. If command destruct packages are included there are two considerations. First, the TSPI must be real time and so will not have the advantage of post-test processing to enhance accuracy. Second, from the range safety point of view, the accuracy problem is open-ended. In other words, range safety personnel can take whatever accuracy can be afforded them by the system and design a safe, effective range with the given accuracy as a constraint.

The equipment configuration used here is the now familiar differential receiver at a surveyed location and a translator on board the tracked vehicle. However, since range safety requirements are real time, the luxury of post-test processing cannot be afforded. Now to estimate a total one sigma UERE for this case the TR 82-2 real time error budget for use of the C/A code (Table VII) can be appropriately modified. This is done by discounting the bias errors, and adding geometric decorrelation, atmospheric uncertainty, and receiver bias errors. Another assumption is required here. The receiver bias error of  $0.5 \sqrt{2}$  for the differential case quoted earlier is for use of the P-code. Based on the difference in chip widths between the P and C/A codes, the receiver bias while using the C/A code is assumed to be  $5.0 \sqrt{2}$ . The results are shown in Table XIII. The new rss is,

therefore, 19.4 meters. Using a PDOP of 2.2 and a filtering improvement factor of 1/3 the final accuracy will be

$$(2.2 \times 19.4) / 3 = 14.2 \text{ meters}$$

Even using the more conservative 90th percentile PDOP value of 3.38 the accuracy is 21.6 meters. This figure should present no problem to test planners and range safety personnel.

Case two involves the much more strict accuracy requirements levied on tracking the missile for test data. In this situation the data can be recorded and later subjected to intensive post-test processing. Drawing from the post-test error budget (Table X) and the errors due to differentiation detailed in the previous chapter, a total UERE for post-test processing can be estimated as shown in Table XIV.

TABLE XIII

Real Time UERE for Use of the C/A Code and the Differential Technique

Error Source	One sigma UERE (meters)
Receiver noise	$2.6 \sqrt{2}$
Receiver bias	$5.0 \sqrt{2}$
Multipath	$12.0 \sqrt{2}$
Geometric decorrelation	0.5
Atmospheric uncertainties	5.0
RSS	19.4

TABLE XIV

Error Budget for Use of the C/A Code and the Differential Technique with  
Post-Test Processing

Error Source	User Equivalent Range Error (1 sigma) meters
Receiver noise	$0.13 \sqrt{2}$
Receiver bias	$0.5 \sqrt{2}$
Multipath	$0.6 \sqrt{2}$
Atmospheric uncertainties	0.5
RSS	1.2

Since the realistic PDOP estimate for the contrived Antigua range was 2.2, the final accuracy is

$$2.2 \times 1.2 = 2.6 \text{ meters}$$

This is within the OR 1508 restriction of 6.1 meter position accuracy.

Concerning velocity accuracies, using the worse case of real time tracking, the accuracy will be within .5 meters per second. This is using a low dynamic receiver and a PDOP value of 3.38. Employing post-mission processing and differential method, the error will be on the order of .01 meters per second. Clearly, the velocity accuracy of the GPS system is well within the OR 1508 requirement.

#### Multipath

At this point it is instructive to further analyze the multipath problem. The multipath error shown in Table XIV is an assumed value

based on the value given in TR 82-2. This was earlier noted to be a dynamic error that is heavily dependent upon geometry. Recall that the time delay in a secondary signal path is

$$\Delta T = (2h/c)\sin \theta$$

and the maximum signal delay for which multipath interference is significant is 1.5 chips.

$$1.5 / 1.023 \times 10^6 \text{ Hz} = 1.467 \times 10^{-6} \text{ seconds} = \Delta T$$

so

$$h = (c\Delta T) / 2\sin\theta = 219.94 / \sin \theta$$

If  $\theta$  corresponds to the mask angle then the height above which multipath at the translator is no longer a factor for a given mask angle is shown in Fig 8.

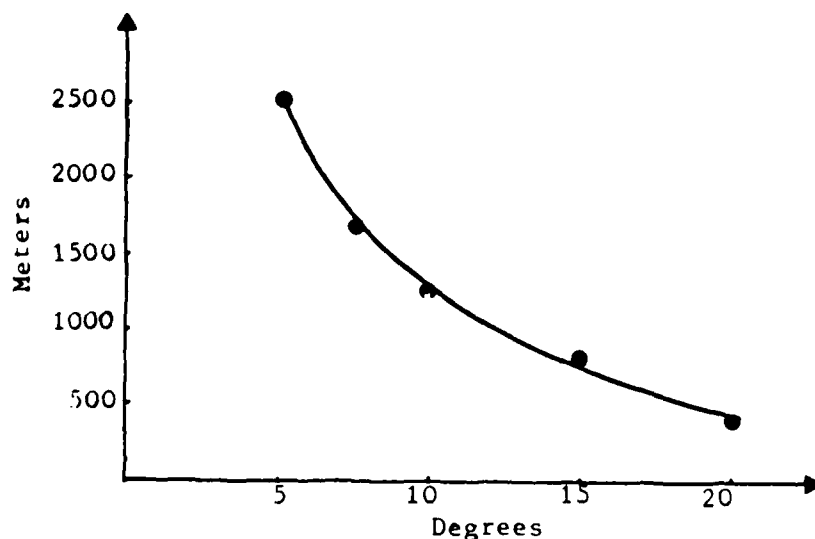


Fig 8. Height vs Mask Angle With No Multipath

In other words, if a mask angle of five degrees is used then the tracked vehicle must not fly below 2524 meters if it is to avoid a loss of accuracy due to multipath. On the other hand, if the mask angle is changed to 20 degrees then the tracked vehicle can fly as low as 643 meters before multipath becomes a problem. Both these values correspond to the look angle between the receiver and satellite being the same as the mask angle.

Using this approach the total UERE from Table XIV can be changed to reflect multipath only at the master receive station assuming that the SRAM II will not fly below the minimum altitude for the particular mask angle. Adopting the previously used value of 0.6 meter but not multiplying by root two since multipath error will occur only at the master receive station, the new rss is 1.1 meters. This is not a large change so there is no great advantage to restricting the test flight envelope of the SRAM II to remain above these minimum heights. If, however, the master receive station could be designed such that multipath of the S-band signal was not a factor the one sigma UERE would be only 0.9 meter. The implications of this analysis will be made clear in the following discussion but first another consideration will be introduced.

A factor that will lower the decision maker's confidence in the 2.6 meter figure from Table XIV is the fact that this is a one sigma figure. The error distribution is assumed to be normal which implies that the one sigma level is one which encompasses 68% of the observations. In other words, 68% of the time the error will be 2.6 meters or less but 32% of the time it will be greater than 2.6 meters.



To calculate the two sigma factor, multiply the one sigma figure by two. This new rss is 5.2 meters and is the accuracy expected 95% of the time. The decision maker can have much more confidence in this estimate and it is still below the maximum 6.1 meter requirement.

Another analysis using multipath may now be accomplished. If the accuracy requirement is 6.1 meters and the two sigma UERE is 2.2 meters (using a one sigma value of 1.1 which corresponds to multipath error only at the master receive station since the tracked vehicle remains above the multipath maximum altitude), the maximum allowable PDOP is  $6.1/2.2=2.77$ . The maximum time intervals over the period of the GPS constellation when the PDOP is no larger than 2.77 are shown in Table XV. The minimum height at which loss of accuracy due to multipath at the tracked vehicle does not occur is also included in this table.

Table XV

Minimum Altitude and Maximum PDOP Window for Mask Angles

Mask Angle (degrees)	Minimum Altitude (meters)	Window for PDOP<2.77 (minutes)
5	2524	135
7.5	1685	120
10.0	1267	95
15.0	850	45
20.0	643	10

This shows that there are significant periods of time when the PDOP can support the minimum accuracy requirement, at higher mask angles. Using higher mask angles decreases the accuracy degradation due to multipath so the advantage is obvious particularly since some SRAM II FOT&E mission profiles can be expected to be at low altitudes, possibly below 1000 meters.

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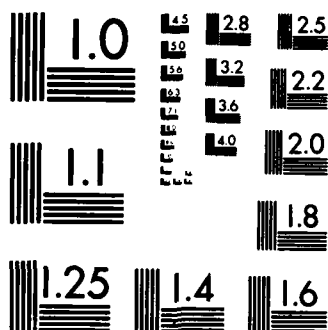
USE OF THE GLOBAL POSITIONING SYSTEM FOR TRAJECTORY  
DETERMINATION OF SRAM. (U) AIR FORCE INST OF TECH  
WRIGHT-PATTERSON AFB OH SCHOOL OF ENGI... E A ZEHNER  
DEC 86 AFIT/GSO/ENS-ENG/86D-2 F/G 17/7

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MICROCOPY RESOLUTION TEST CHART  
NATIONAL BUREAU OF STANDARDS-1963-A

## VII. Discussion and Recommendation

The four research questions posited in Chapter I have now all been answered. To review, the structure and use of GPS was detailed with the discussion of the hardware (receivers, ground antennas, monitor stations, the master control station, and the satellites) and the system working concepts. The limits to the accuracy of the system in general and as they apply to the proposed configuration for SRAM II tracking were documented. These limits were attributed to the satellite signal delay sources such as the atmosphere, the transmitters and receivers. In addition, losses of accuracy occur due to clock synchronization limitations, multipath, ephemeris errors and tracked vehicle dynamics were discussed.

From this information the accuracy of the GPS system was calculated. Then the use of differentiation, HDLV equipment, smoothing, and post-test processing to improve the accuracy was introduced. Finally, the question concerning the expected accuracy of the GPS system when used to track SRAM II missiles was addressed. The accuracy of the GPS system using the C/A code, differentiation, translation, and post-test processing was determined.

Even though the one sigma UERE for post-test processing and use of the C/A code with the differential technique is 2.6 meters, only about half that required by the operations requirements, there are some additional considerations that make this figure less attractive. These and some final thoughts will be discussed in this section.

## Multipath

The figures used for multipath in this and the previous chapters were generalized due to the complex nature of multipath. The amount of error due to multipath is a function of the angle between the local horizontal at the GPS receiver and the line of sight to the satellite, the distance between the receiver and the reflecting surface, the characteristics of the receive antenna, the additional path length of the indirect signal, and the relative magnitudes of the indirect and direct signals. This last factor has a strong dependence upon the reflectance of nearby surfaces at the appropriate frequencies.

All the above factors except the receive antenna characteristics will be changing constantly during the missile test flight. In addition, the multipath characteristics at the translator and at the master receive station are entirely different. The TR 82-2 figure for multipath for the differential case using the C/A code was obtained by multiplying the non-differential C/A code multipath estimate by root two. This assumes the multipath will be the same at both receivers which is obviously not the case. An accurate estimate of the multipath error must consider the translator and the master receive station separately.

This makes quantification of the actual multipath very difficult and time consuming. It is strongly recommended that a follow-on study using a more intensive analytical or a simulation approach be accomplished to obtain multipath error estimates with which the decision maker can be confident before a final decision is made.

### Future Accuracy Requirement

Compounding the multipath problem is the possibility of a more strict accuracy requirement for the SRAM II. The SRAM II is to be an improvement over the previous SRAM and greater accuracies are to be expected. In fact, Capt Barry Tanner, the SRAM II Test Manager at Aeronautical Systems Division stated that the requirement may be as low as ten feet (about 3 meters) for the new missile. The two sigma UERE is 5.2 meters so if this is the case, GPS cannot supply the necessary accuracy.

In fact, even if multipath is discounted entirely by using a higher mask angle so there would be no significant multipath interference at flight altitudes, GPS would not be accurate enough. In previous sections it was established that the one sigma UERE corresponding to a mask angle of 20 degrees is 0.9 meters. The two sigma figure would then be 1.8 meters. At a mask angle of 20 degrees the PDOP at the Antigua range never drops low enough to yield three meter accuracy. This is because the PDOP will increase as the mask angle is increased. Continuing the analysis for other mask angles it was found that there are no values for the mask angle with their respective PDOPs that will have three meter accuracy.

### Dynamics

Another error source not adequately addressed is dynamics. Recall that maneuvers subjecting the receiver to accelerations of five g's or more can severely distort the phase displacement of the incoming PN coded signals. This introduces error into the range measurement but much of the error can be removed using the Jet Propulsion Laboratory's

HDLV equipment. However, the amount of error removed cannot be adequately estimated here. The JPL documentation listed accuracy figures for use of the P-code but not the C/A code. Further testing of the HDLV using the C/A code should be accomplished to obtain an accurate error estimate. It is possible that the dynamic error could be a dominant factor in the total UERE so it is important that this research is accomplished before a final decision is made as well.

#### Cooperative Tracking

Still, there are ways of further improving the accuracy of the GPS system. The first of these is to use the system in concert with another tracking system, such as the cinetheodolite. This is a high speed fixed camera for single point position determination. While it would not be used to track the entire test flight, it could be used to obtain a definite position fix on the missile at the time of its fuzing. This is the portion of the test flight which simulates hitting the target. The system is capable of two to three foot accuracy so this critical portion of the test flight could be scored very accurately.

The advantage of the cinetheodolite does not stop at fuzing. Recall that the advantage of using the differential technique is the correction for the bias errors (SV clock and ephemeris errors, group delay, ephemeris prediction and model implementation, and ionospheric and tropospheric delay compensation). A disadvantage is that a new term, receiver bias, must then be added in to account for the difference between the differential receiver and the receiver/translator package used for translation. By accurately finding the position of the missile independently of the GPS receivers, the effect of this new receiver bias



term can be negated. Furthermore, since the translated GPS signals from the entire test flight were recorded, the improvement in accuracy at fuzing can be backtracked to improve the position accuracy all along the test flight path. Removing the receiver bias term from Table XIV, the new one sigma UERE would be 1.0 meter and the accuracy (UERE X PDOP) would be 2.2 meters.

#### P-Code Translation

Another means of improving the accuracy of the GPS system for this application is use of the P-code. The P-code is inherently more accurate than the C/A code but was not used in development of the current translators due to the high bandwidth requirements. If a solution to the bandwidth problem is found the next generation of translators could operate with the P-code. This would result in a tremendous increase in accuracy. Drawing from the TR 82-2 post-test error budget for the P-code and making the familiar adjustments for the differential case, the one sigma UERE would be 0.1 meters so the accuracy would be 0.2 meters. In addition, this figure would be far less susceptible to degradation due to multipath errors.

#### Recommendation

The GPS system is found to be a viable prospect for a SRAM II tracking system. It meets the 6.1 meter accuracy requirement and is well within the standard for velocity measurement. However, it is very important to note that these figures were made under assumed values for geometric decorrelation, atmospheric uncertainties as they effect differential operations, and receiver bias. Of greater importance are the poorly quantified multipath error contribution and the effect of

dynamics on total accuracy. Further research is required to adequately address these problems. A final consideration is the GPS system using the C/A code does not meet the tighter accuracy requirements under consideration for SRAM IIs.

Therefore, the author does not recommend use of the GPS system to track SRAM II missiles at this time. Nevertheless, the system deserves further attention. If the multipath and dynamic error problems can be better quantified and are shown not to force the error above tolerable limits then the GPS system could be used. If P-code translators are developed then the GPS system would offer accuracy over an order of magnitude better than what is currently required.

### Appendix: PDOP Data

PDOPs for the representative Antigua range for 12 hours from 10:00 a.m. on 1 June 1987 at 17.63° North 61.0° West and 500 meters altitude using an 18 satellite constellation and a mask angle of 5°.

PDOP	PTIME(HR)	PDOP	PTIME(HR)
2.825007	10.000000	2.371914	13.583333
2.751261	10.083333	2.413898	13.666667
2.465070	10.166667	2.477282	13.750000
2.482231	10.250000	3.244576	13.833333
2.512739	10.333333	3.382298	13.916667
2.557242	10.416667	3.529042	14.000000
2.493581	10.500000	3.680302	14.083333
2.120461	10.583333	3.830013	14.166667
2.174312	10.666667	3.970510	14.250000
2.225584	10.750000	2.578361	14.333333
2.164432	10.833333	2.592931	14.416667
2.235482	10.916667	2.641822	14.500000
2.289646	11.000000	2.724935	14.583333
2.284496	11.083333	2.844365	14.666667
2.292142	11.166667	3.004451	14.750000
2.310241	11.250000	2.869684	14.833333
2.336753	11.333333	2.717977	14.916667
2.369808	11.416667	2.595490	15.000000
2.407608	11.500000	2.498666	15.083333
2.423912	11.583333	2.424855	15.166667
2.337660	11.666667	2.372295	15.250000
2.531486	11.750000	3.086299	15.333333
2.534516	11.833333	2.897042	15.416667
2.537601	11.916667	2.737053	15.500000
3.980729	12.000000	2.369833	15.583333
3.836093	12.083333	2.438830	15.666667
2.540877	12.166667	2.528285	15.750000
2.565195	12.250000	2.540279	15.833333
2.578932	12.333333	2.504540	15.916667
2.496070	12.416667	2.482575	16.000000
2.420738	12.500000	2.473857	16.083333
2.367019	12.583333	2.478104	16.166667
2.334037	12.666667	2.495321	16.250000
2.321868	12.750000	2.054932	16.333333
2.331689	12.833333	2.085789	16.416667
2.366058	12.916667	2.132354	16.500000
3.313955	13.000000	2.195005	16.583333
3.354568	13.083333	2.274773	16.666667
2.262181	13.166667	2.330406	16.750000
2.310749	13.250000	2.504379	16.833333
2.359128	13.333333	2.513257	16.916667
2.351887	13.416667	2.497566	17.000000
2.351006	13.500000	2.517554	17.083333

PDOP	PTIME(HR)
2.571041	17.166667
2.658225	17.250000
2.781452	17.333333
3.486527	17.416667
3.720723	17.500000
3.960176	17.583333
3.801482	17.666667
3.633683	17.750000
2.623243	17.833333
2.613622	17.916667
2.578692	18.000000
2.542400	18.083333
2.505601	18.166667
2.089888	18.250000
2.083793	18.333333
2.087541	18.416667
2.100725	18.500000
2.123075	18.583333
2.127811	18.666667
2.087882	18.750000
2.069783	18.833333
2.225230	18.916667
2.157486	19.000000
2.106143	19.083333
2.070620	19.166667
2.050995	19.250000
2.048078	19.333333
2.063559	19.416667
2.100281	19.500000

PDOP	PTIME(HR)
2.092607	19.583333
2.070433	19.666667
2.057310	19.750000
2.484634	19.833333
2.408776	19.916667
2.351861	20.000000
2.847012	20.083333
3.060556	20.166667
3.228987	20.250000
3.386157	20.333333
3.552027	20.416667
3.719836	20.500000
2.742823	20.583333
2.889464	20.666667
2.993038	20.750000
2.822949	20.833333
2.694448	20.916667
2.602616	21.000000
2.544817	21.083333
2.520561	21.166667
2.531694	21.250000
2.507151	21.333333
2.441907	21.416667
2.390805	21.500000
2.352786	21.583333
3.223108	21.666667
3.092901	21.750000
2.976925	21.833333
2.876577	21.916667

PDOPs for the Ramey AFB Air Weather Station site for 12 hours beginning at 10:00 a.m. on 1 June 1987 at 18.6° North 67.55° West and 500 meters altitude using an 18 satellite constellation and a mask angle of 5°.

PDOP	PTIME(HR)
2.481624	10.000000
2.470952	10.083333
2.473198	10.166667
2.488484	10.250000
2.517199	10.333333
2.559960	10.416667
2.489835	10.500000
2.409943	10.583333
2.683055	10.666667
2.807060	10.750000
3.001376	10.833333
3.152648	10.916667
2.392447	11.000000
2.454424	11.083333
2.538354	11.166667
2.646383	11.250000
2.781543	11.333333

2.947784	11.416667
2.932969	11.500000
2.791871	11.583333
2.689252	11.666667
2.621788	11.750000
2.588342	11.833333
2.590057	11.916667
3.916329	12.000000
3.771533	12.083333
3.621051	12.166667
3.471831	12.250000
3.329215	12.333333
2.520129	12.416667
2.444177	12.500000
2.390164	12.583333
2.357234	12.666667
2.345495	12.750000

PDOP	PTIME(HR)
2.356166	12.833333
2.346273	12.916667
2.297603	13.000000
3.352380	13.083333
3.325073	13.166667
2.346244	13.250000
2.355473	13.333333
2.332184	13.416667
2.332249	13.500000
2.353754	13.583333
2.396026	13.666667
2.459416	13.750000
2.545193	13.833333
2.560866	13.916667
2.542493	14.000000
3.730720	14.083333
3.879329	14.166667
4.018237	14.250000
4.138458	14.333333
2.632439	14.416667
2.680556	14.500000
2.379941	14.583333
2.425341	14.666667
2.386228	14.750000
2.350783	14.833333
2.320789	14.916667
2.298066	15.000000
2.284547	15.083333
2.282392	15.166667
2.292809	15.250000
2.205773	15.333333
2.257507	15.416667
2.193505	15.500000
2.662407	15.583333
2.447158	15.666667
2.536420	15.750000
2.536620	15.833333
2.498662	15.916667
2.474398	16.000000
2.463295	16.083333
2.776905	16.166667
2.858188	16.250000
2.956048	16.333333
3.069657	16.416667
3.197739	16.500000
2.343893	16.583333
2.379943	16.666667
2.428893	16.750000
2.491762	16.833333
2.526903	16.916667
2.510301	17.000000
2.529460	17.083333

PDOP	PTIME(HR)
2.582167	17.166667
2.668600	17.250000
2.791085	17.333333
2.954193	17.416667
2.930697	17.500000
3.910669	17.583333
3.752917	17.666667
3.586482	17.750000
3.420516	17.833333
3.262140	17.916667
3.106484	18.000000
2.880270	18.083333
2.731947	18.166667
2.409698	18.250000
2.061796	18.333333
2.064201	18.416667
2.076025	18.500000
2.096981	18.583333
2.112200	18.666667
2.071816	18.750000
2.053240	18.833333
2.053423	18.916667
2.070523	19.000000
2.103615	19.083333
2.152516	19.166667
2.067434	19.250000
2.064399	19.333333
2.079825	19.416667
2.116539	19.500000
2.121310	19.583333
2.098116	19.666667
2.084068	19.750000
2.079424	19.833333
2.468041	19.916667
2.504611	20.000000
2.541664	20.083333
2.578370	20.166667
3.271894	20.250000
3.429285	20.333333
3.595027	20.416667
3.762188	20.500000
3.921737	20.583333
3.735616	20.666667
3.500608	20.750000
2.800845	20.833333
2.672398	20.916667
2.580484	21.000000
2.522435	21.083333
2.497729	21.166667
2.508163	21.250000
2.507197	21.333333
2.441123	21.416667

PDOP	PTIME(HR)
2.389176	21.500000
2.350267	21.583333
2.141168	21.666667
2.092601	21.750000
2.059755	21.833333
2.493984	21.916667

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